QC1

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# Introduction to Quantum Computing

– Main Principles of Quantum Mechanics according to Copenhagen Interpretation

1. Quantum Reality exists in the Hilbert Space which is complex vector space

2. There are Collapsed quantum States in Hilbert Space $|\lambda_n\rangle$ possessing the properties $\lambda_n$ such that $\langle\lambda_n|\lambda_m\rangle = \delta_{mn}$

3. For the set of Collapsed states one can construct operator $\hat{A}$ such that $\hat{A}|\lambda_n\rangle = \lambda_n|\lambda_n\rangle$

4. For any quantum state $|\psi\rangle$ one can expand it through the collapsed states

$$|\psi\rangle = \sum_{n=1}^{\infty} \alpha_n|\lambda_n\rangle$$

where $\alpha_n = \langle\lambda_n|\psi\rangle$

and $|\alpha_n|^2$ – gives the probability that while measuring state $|\psi\rangle$ one obtains the property $\lambda_n$

– Copenhagen Interpretation states that after such measurement state $|\psi\rangle$ collapses to $|\lambda_n\rangle$ and further measurements with the same device will yield $|\lambda_n\rangle$ – all the time

5. For two noninteracting Quantum States $|\psi_1\rangle$ and $|\psi_2\rangle$ one has

$$|\psi_{12}\rangle = |\psi_1\rangle|\psi_2\rangle = \sum_{n=1}^{\infty}\alpha_n|\lambda_n\rangle \sum_{m=1}^{\infty}\alpha_m|\lambda_m\rangle$$

6. For k noninteracting quantum state

$$|\psi_{1\ldots k}\rangle = \prod_{i=1}^{k}\sum_{n_i=1}^{\infty}\alpha_{n_i}|\lambda_{n_i}\rangle$$

Example of two state quantum particle.

$|\lambda_n\rangle$ where $n=1,2$ (two state system)

$$\boxed{\text{Say } \left.\begin{matrix} n=1 \equiv + \\ n=2 \equiv - \end{matrix}\right\} \text{Charges}}$$

1. One Particle Case

$$|\psi_1\rangle = \sum_{n=1}^{2}\alpha_n|\lambda_n\rangle = \alpha_+|\lambda_+\rangle + \alpha_-|\lambda_-\rangle$$

$$|\alpha_+|^2 + |\alpha_-|^2 = 1$$

- probability amplitude that the measurement yields (+) charge

$$\langle\lambda_+|\psi_1\rangle = \alpha_+$$

- Probability amplitude that the measurement yields (−) charge

$$\langle\lambda_-|\psi_1\rangle = \alpha_-$$

- Probability that measurement yields any charge

$$|\alpha_+|^2 + |\alpha_-|^2 = 1$$

- if outcome of + and − charges equally probable

$$\alpha_+ = \alpha_- = \sqrt{\tfrac{1}{2}}$$

2. Two noninteracting Particle Case

2) Two noninteracting particle case

2) Two

(unrestricted total charge)

$$|\psi_{12}\rangle = |\psi_1\rangle|\psi_2\rangle = \sum_{n=1}^{2}\alpha_n^{(1)}|\lambda_n^{(1)}\rangle \sum_{m=1}^{2}\alpha_m^{(2)}|\lambda_m^{(2)}\rangle = //$$

$$// = \left(\alpha_+^{(1)}|\lambda_+^{(1)}\rangle + \alpha_-^{(1)}|\lambda_-^{(1)}\rangle\right)\left(\alpha_+^{(2)}|\lambda_+^{(2)}\rangle + \alpha_-^{(2)}|\lambda_-^{(2)}\rangle\right) =$$

$$= \alpha_+^{(1)}\alpha_+^{(2)}|\lambda_+^{(1)}\rangle|\lambda_+^{(2)}\rangle + \alpha_-^{(1)}\alpha_-^{(2)}|\lambda_+^{(1)}\rangle|\lambda_-^{(2)}\rangle +$$

$$\alpha_-^{(1)}\alpha_+^{(2)}|\lambda_-^{(1)}\rangle|\lambda_+^{(2)}\rangle + \alpha_-^{(1)}\alpha_-^{(2)}|\lambda_-^{(1)}\rangle|\lambda_-^{(2)}\rangle$$

Note that Always $\langle\psi_{12}|\psi_{12}\rangle = 1$ (closure theore

$\langle\lambda_+|\lambda_-\rangle = 0$

$\langle\lambda_+|\lambda_+\rangle = 1$

– Probability amplitude that particle 1 has a positive charge. negative charge

$$\langle\lambda_+^{(2)}|\langle\lambda_+^{(1)}|\psi_{12}\rangle = \alpha_+^{(1)}\alpha_+^{(2)}|\lambda_+^{(2)}\rangle + \alpha_+^{(1)}\alpha_-^{(2)}|\lambda_-^{(2)}\rangle$$

- Suppose the measurement was done and the above was the outcome
- for the example in which + or – outcomes of one particle measurement are equal

$$\langle\lambda_+^{(2)}|\langle\lambda_+^{(1)}|\psi_{1,2}\rangle = \frac{1}{2}|\lambda_+^{(2)}\rangle + \frac{1}{2}|\lambda_-^{(2)}\rangle$$

$\langle\lambda_-^{(2)}|$

– what is the probability amplitude that the measurement of the second particle charge yeds + or – value

$>$ for(+) $\langle\lambda_+^{(2)}|\langle\lambda_+^{(1)}|\psi_{12}\rangle = \boxed{\frac{1}{2}}\langle\lambda_+^{(2)}|\lambda_+^{(2)}\rangle + \boxed{\frac{1}{2}}\underbrace{\langle\lambda_+^{(2)}|\lambda_-^{(2)}\rangle}_{=0} = \frac{1}{2}$

fo(–) $\langle\lambda_-^{(2)}|\langle\lambda_+^{(1)}|\psi_{12}\rangle = \frac{1}{2}\underbrace{\langle\lambda_-^{(2)}|\lambda_+^{(2)}\rangle}_{=0} + \frac{1}{2}\underbrace{\langle\lambda_-^{(2)}|\lambda_-^{(2)}\rangle}_{1} = \frac{1}{2}$

– two outcomes of (2) particle charge measurement are equal. ] !

– the same result will be also if the measurement of the (1) particle yieldes (–) charge

$$\langle\lambda_-^{(1)}|\psi_{12}\rangle = \alpha_-^{(1)}\alpha_+^{(2)}|\lambda_+^{(2)}\rangle + \alpha_-^{(1)}\alpha_-^{(2)}|\lambda_-^{(2)}\rangle$$

– measuring the charge of the second particle

for (+) outcome $\langle\lambda_+^{(2)}|\langle\lambda_-^{(1)}|\psi_{12}\rangle = \alpha_-^{(1)}\alpha_+^{(2)} = \frac{1}{2}$

for (–) utcom $\langle\lambda_-^{(2)}|\langle\lambda_-^{(1)}|\psi_{12}\rangle = \alpha_-^{(1)}\alpha_-^{(2)} = \frac{1}{2}$

– again two outcomes of particle (2) measurements of charge are equal.

Thus if total charge of the (1,2) system is not fixed, the outcome of the measurement of the second charge does not depend on the outcome of the measurement of the first charge.

... are not – entangled

particle 1 and 2 are entangled

2.' Two noninteracting Particle Case in which total charge is 0

$$\langle\lambda_+^{(1)}|\Psi_{12}\rangle = N(\alpha_+^{(1)}\alpha_-^{(2)}|\lambda_+^{(1)}\rangle|\lambda_-^{(2)}\rangle + \alpha_-^{(1)}\alpha_+^{(2)}|\lambda_-^{(1)}\rangle|\lambda_+^{(2)}\rangle)$$

$$\langle\Psi_{12}|\Psi_{12}\rangle = 1 \quad N[\alpha_+^{(1)}\alpha_-^{(2)}\cancel{\langle\lambda_+^{(1)}|\lambda_+^{(1)}\rangle}|\lambda_-^{(2)}\rangle + \alpha_-^{(1)}\alpha_+^{(2)}\cancel{\langle\lambda_+^{(1)}|\lambda_-^{(1)}\rangle}|\lambda_+^{(2)}\rangle]$$

→ Probability amplitude that particle 1 has a positive charge

$$\langle\lambda_+^{(2)}\langle\lambda_+^{(1)}|\Psi_{12}\rangle = N\,\alpha_+^{(1)}\alpha_-^{(2)}|\lambda_-^{(2)}\rangle$$

$\langle x|$

→ Probability amplitude that measurement of the charge of the (2) particle yields + or −

$$\text{for}(+)\quad \langle\lambda_+^{(2)}|\langle\lambda_+^{(1)}|\Psi_{12}\rangle = N\alpha_+^{(1)}\alpha_-^{(2)}\langle\lambda_+^{(2)}|\lambda_-^{(2)}\rangle = 0$$

$$\text{for}(-)\quad \langle\lambda_-^{(2)}|\langle\lambda_+^{(1)}|\Psi_{12}\rangle = N\alpha_+^{(1)}\alpha_-^{(2)}\cancel{\langle\lambda_-^{(2)}|\lambda_-^{(2)}\rangle} = N\alpha_+^{(1)}\alpha_-^{(2)} \neq 0$$

Thus if the measurement of the charge of the first particle yields (+)
The measurement of the second particle will yield only (−)

Particle 1 and 2 are entangled

# Qubits.

– Classical bit is a device that can have two values (1) or (0) → Electrical switch is such a device

– for one bit one can store an information 1 or 0 (+ −)

– adding another bit 11 or 10 or 01 or 00 ++ or +−
4×

– for 3 bits 111 or 110 or 101 8 possible
8

– 7-bits – byte 7 combination of 1 and 0'

– n – bits — can store n – length information of 1 and 0s

$2^n$ bits $2^2$

⟹ Quantum Bit – Qubit

– Consider particle that can be in two (+) or (–) quantum states

– One such particle is called qubit

1 particle $|\psi_1\rangle = \alpha_+^{(1)}|\lambda_+^{(1)}\rangle + \alpha_-^{(1)}|\lambda_-^{(1)}\rangle$ — qubit

$\boxed{+}$ and $\boxed{-}$

1 0

2. particles $|\psi_{12}\rangle = |\psi_1\rangle|\psi_2\rangle = \alpha_+^{(1)}\alpha_+^{(2)}|\lambda_+^{(1)}\rangle|\lambda_+^{(2)}\rangle + \alpha_+^{(1)}\alpha_-^{(2)}|\lambda_+^{(1)}\rangle|\lambda_-^{(2)}\rangle +$

$+ \alpha_-^{(1)}\alpha_+^{(2)}|\lambda_-^{(1)}\rangle|\lambda_+^{(2)}\rangle + \alpha_-^{(1)}\alpha_-^{(2)}|\lambda_-^{(1)}\rangle|\lambda_-^{(2)}\rangle$

2-qubits $2^2$ information 1 1 ++, and 1 0 +–, and 0 1 –+, or 0 0 ––

needs 4 – bits for same amount of information

– switches

3 – qubits $2^3$ information

needs 8 – bits

$\boxed{2^{2^{n\text{-qubits}}}} \approx N_{inf}$

300 – qubits $2^{300}$ information

$\boxed{N_f \sim n \text{ bits}}$

$2^{300} \approx$ needs $2.04 \times 10^{90}$ – classical bits switches

number of atoms in the Universe $10^{82}$

bits $\sim 2 \cdot \times 10^{90}$ – classical computer