QC11

Wednesday, March 10, 2021 6:35 PM

Reversible Computation

Invertible.

2 - inputs → 1 output

AND

Input		Output
P	Q	
0	0	0
0	1	0
1	0	0
1	1	1

Half Adder.

Input		Output	
P	Q	digit	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

John von Neumann

information lost - energy is expended

Reversable Gates: CNOT, Toffoli, Fredkin

⇒ Controlled Not Gate → CNOT

2 - inputs → 2 - outputs

1st → control bit

x

0 - no effect on 2nd bit

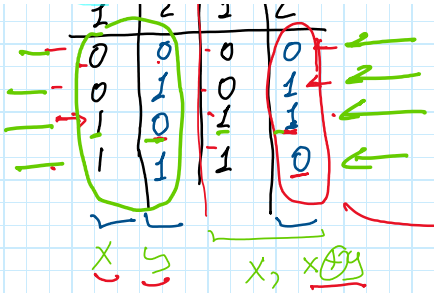
1 - it acts like the Not gate on the second bit

2nd → y

input		output	
x	y	x	x+y

$x \oplus y$

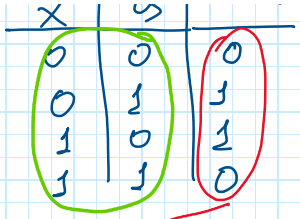
Invertible



x, y

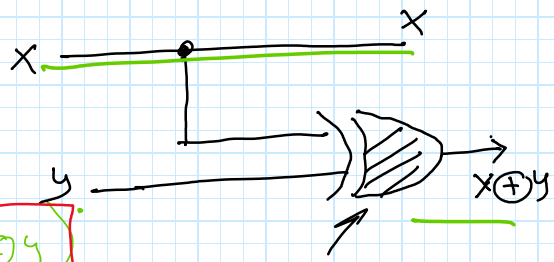
x, x⊕y

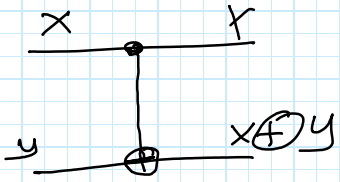
exclusive or ⊕



$f(x,y) = (x, x \oplus y)$

inputs





usual picture

CNOT is its own inverse

CNOT

Input

Input		Output	
x	y	x	x⊕y
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

CNOT

INPUT		OUTPUT	
x'	y'	x'	x'⊕y'
0	0	0	0
0	1	0	1
1	1	1	0
1	0	1	1

logical Tables

CNOT is its own inverse

CNOT · CNOT = I

$\Leftrightarrow \quad (x,y) \to f(x,y) = (x, x \oplus y)$ CNOT

$f(x, x \oplus y) = f(x, x \oplus x \oplus y) = (x,y)$

y' y'

$x \oplus x \oplus y = (y) \Rightarrow$

Using the Fact That $x \oplus x = 0$ $\quad 0 \oplus y = y$

$\Rightarrow$ The Toffoli Gate: Tommaso Toff

3-inputs 3 outputs

3- inputs

1 → 1st input

2 → 2nd input

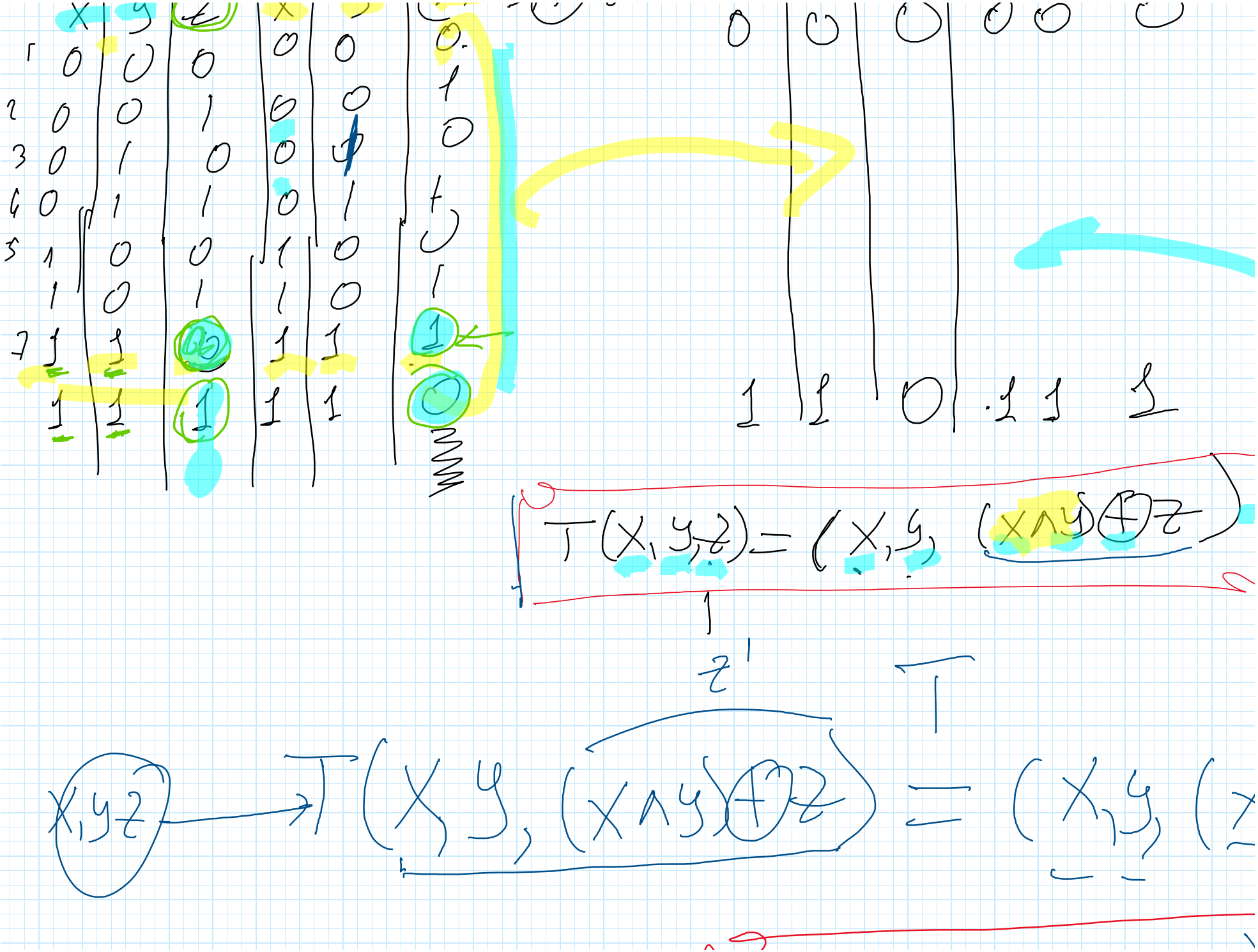
} control bits

3rd-bit is flipped — called also CCNOT

otherwise 3rd-bit stays the same

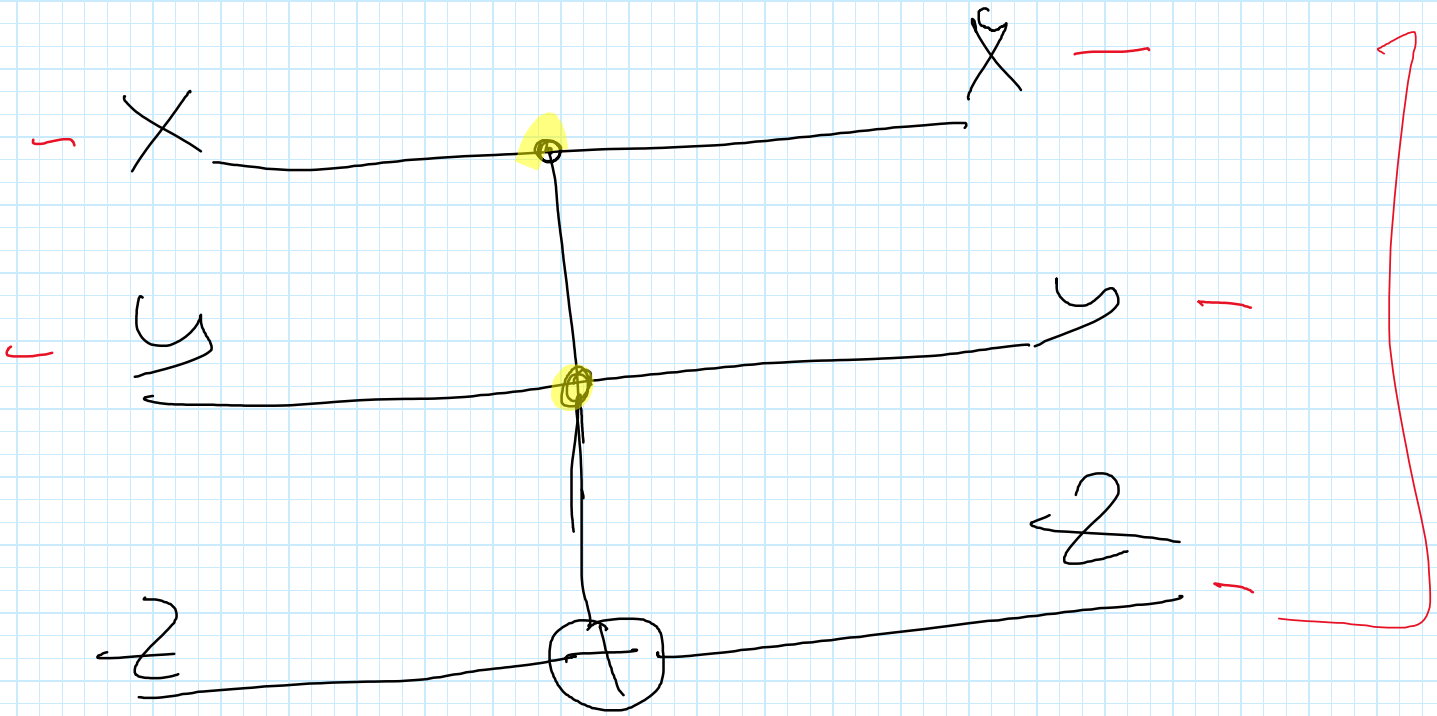
TG

input			output		


T(X, Y, Z) = (X, Y, (X ∧ Y) ⊕ Z)
(X, Y, Z) → T(X, Y, (X ∧ Y) ⊕ Z) = (X, Y,

$(x \wedge y) \oplus (x \wedge y$

using



To

NAND

$f(x,y) =$

$\neg(x \wedge y) =$

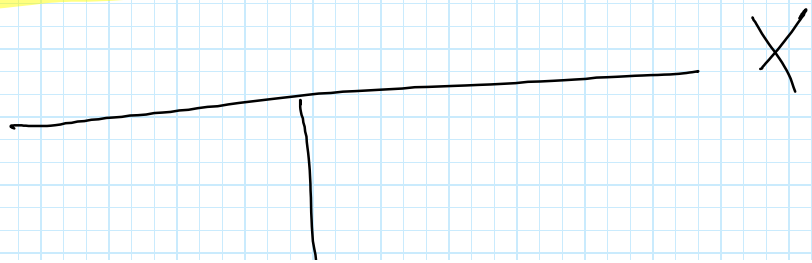
$\neg(0 \wedge 0)$

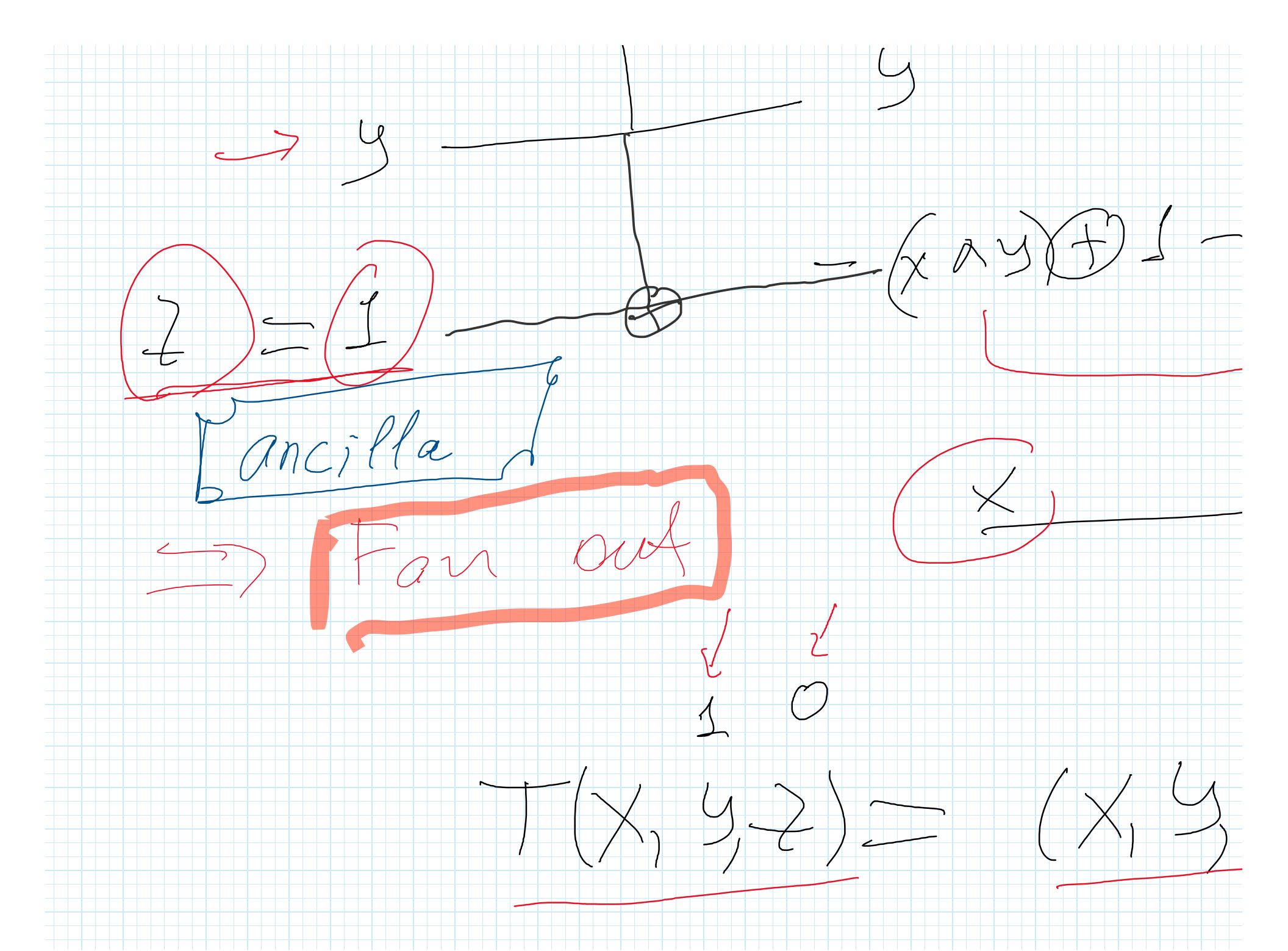
$\neg 0$

1

$T(x, y, 1) =$

$\Rightarrow x$




y
z = 1
(x ∧ y) ⊕ z
Ancilla
Fan out
x
1
0
T(x, y, z) = (x, y

$$T(x, 1, 0) = (x,$$

$$(x \wedge 1) \oplus 0$$

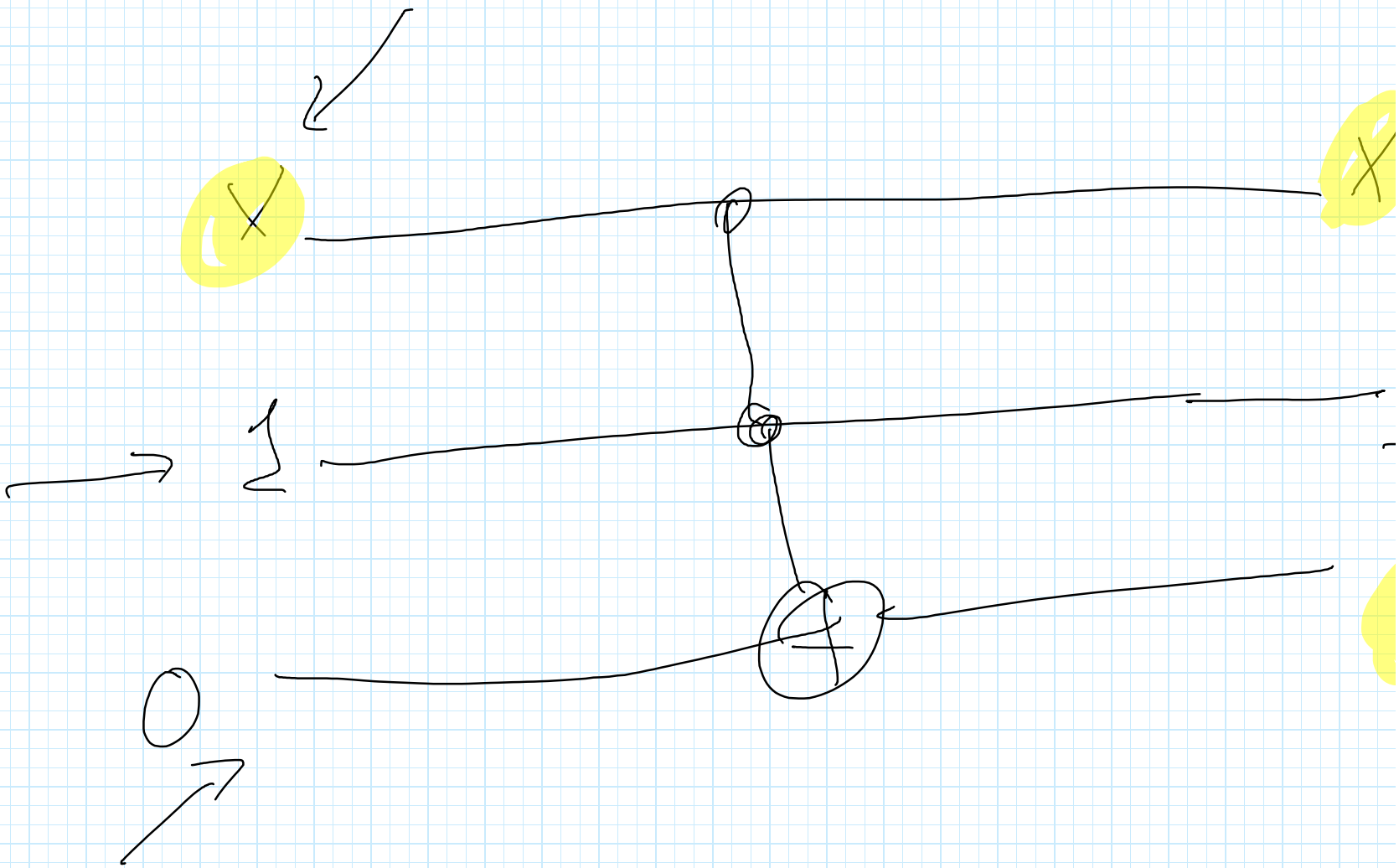
$$(0 \wedge 1)$$

$$0 \oplus 0 =$$

$$\times$$

$$(1 \wedge 1)$$

$$1 \oplus 0$$



Any Boolean circuit

using just Toffoli gate

ancilla bits

garbage bits