QC12

Monday, March 15, 2021 4:31 PM

The Fredkin Gate

- Three inputs and three outputs
  1st - input is a control bit — 0 (2nd, 3rd inputs) are not changed; — 1 (2nd, 3rd inputs) swap (3rd, 2nd inputs)

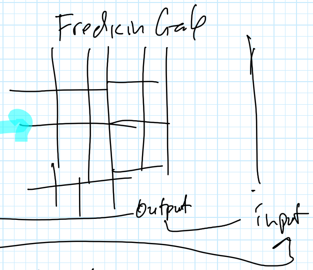
$$\begin{cases} F(0,y,z) = (0,y,z) \\ F(1,y,z) = (1,z,y) \end{cases}$$

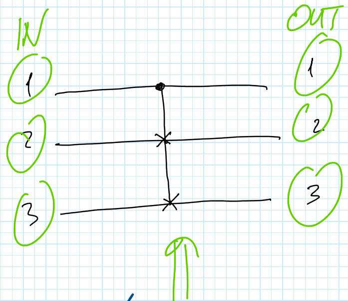
- Logical / Truth table

Fredkin Gate

Input x	y	z	Output x		
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	0	1
1	1	1	1	1	1

input





- Fredkin Gate is invertible. V

+ Number of 1's conserved

Knew NAND

- It is its own inverse V

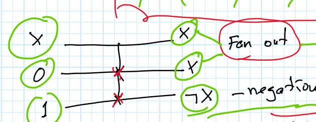
- Notice $F(0,0,1) = (0,0,1)$ and $F(1,0,1) = (1,1,0)$

$x\ x\ \neg x$ $\quad$ $x\ \neg x$

$F(x,0,1) = (x, x, \neg x)$ → Fan out → negation

ancilla

$$F(X,0,1) = (X,X,\neg X)$$



Negator

- Set $z=0$

$F(0,0,0) = (0,0,0)$, $\quad F(0,1,0) = F(0,1,0)$ $\quad F(1,1,0) = (1,0,1)$

$1 \wedge 1 = 0$ $\quad$ $(x \wedge y)$ $\quad$ $\neg x \wedge y$ $\quad$ $1 \wedge 1$ $\quad$ $\neg x \wedge y$ $\quad$ $x \wedge y$

$F(x,y,0) = (x, \neg x \wedge y, x \wedge y)$ — AND

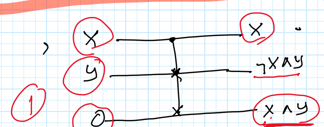
(2) → $\neg x \wedge y$

(3) → $x \wedge y$

NAND

$$F(X,y,0) = (X, \neg X \wedge y, X \wedge y)$$

- Fredkin Gate is universal,

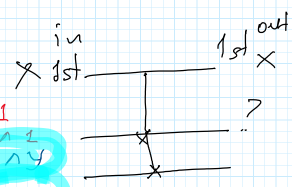


AND

- → $F(0,y,z) = (0,y,z)$, $F(1,y,z) = (1,z,y)$
  - 1st output number is always equals to 1st input X
  - 2nd output will be 1 if in the input $x=0$ and $y=1$ or $y=1$, and $z=1$

$1 \wedge 1$ $\quad$ $\neg x \wedge y$ $\quad$ $x \wedge z$

→ $(\neg x \wedge y) \vee (x \wedge z)$



0

- 2nd output will be $\boxed{0}$, if in the input $x=0$ and $y=0$ $\quad \neg x \wedge y \quad 1 \wedge 0$

or $x=1$ and $z=0$ $\quad x \wedge z \quad 1 \wedge 0 \to 0$

2nd $\boxed{(\neg x \wedge y) \vee (x \wedge z)}$

- 3rd output will be $\boxed{1}$, if the input $x=0$ and $z=1$ $\quad \neg x \wedge z \quad 1 \wedge 1 \to 1$

or $x=1$ and $y=1$ $\quad x \wedge y \to 1 \wedge 1 \to 1$

$\boxed{(\neg x \wedge z) \vee (x \wedge y)}$

- 3rd output will be $\boxed{0}$, if the input $x=0$ and $z=0$ $\quad \neg x \wedge z$

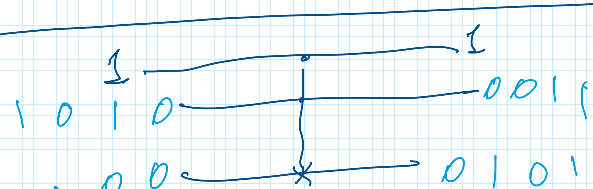
or $x=1$ and $y=0$ $\quad x \wedge y$

3rd $\boxed{(\neg x \wedge z) \vee (x \wedge y)}$

$\Rightarrow$ All together

$$\boxed{F(x,y,z) = (x, (\neg x \wedge y) \vee (x \wedge z), (\neg x \wedge z) \vee (x \wedge y))}$$

$$P \vee Q = \neg(\neg P \wedge \neg Q)$$



NAND

P	Q	
1	1	0
1	0	1
0	1	1
0	0	1