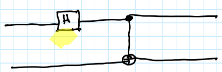
QC15

Monday, March 29, 2021 6:20 PM

The Bell Circuit

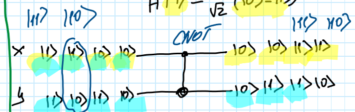
CNOT



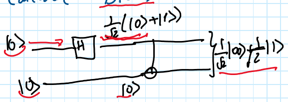
$\boxed{\hat{B}}$

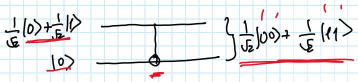
$\hat{H}|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$!

$\hat{H}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$



Calculate $B|00\rangle$

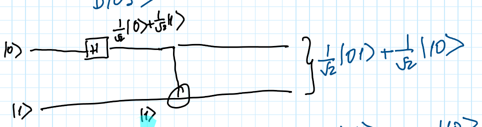




So

$$\boxed{B|00\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle}$$!

$B|01\rangle$



$$CNOT\left(\frac{1}{\sqrt{2}}|0\rangle|1\rangle + \frac{1}{\sqrt{2}}|1\rangle|1\rangle\right) = \frac{1}{\sqrt{2}}\overset{|01\rangle}{|0\rangle|1\rangle} + \frac{1}{\sqrt{2}}\overset{|10\rangle}{|1\rangle|0\rangle}$$

$$\boxed{\underbrace{\hat{B}B}_{I}|01\rangle = \hat{B}\left(\frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle\right)} \Rightarrow |01\rangle = \hat{B}\left(\frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle\right)$$

$B^{-1} = B$

$$\boxed{B|10\rangle = \frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle} \qquad |10\rangle = \hat{B}\left(\frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle\right)$$

$$\boxed{B|11\rangle = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle} \qquad |11\rangle = \hat{B}\left(\frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle\right)$$

$B \quad |00\rangle \quad |01\rangle \quad |10\rangle \quad |11\rangle$ OK

$\langle 01|10\rangle = 0$ $\quad \langle 01|01\rangle = 1$

Orthogonal Basis

$$\underbrace{\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle}_{\lambda_1}, \quad \underbrace{\frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle}_{\lambda_2}, \quad \underbrace{\frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle}_{\lambda_3}, \quad \underbrace{\frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle}_{\lambda_4}$$

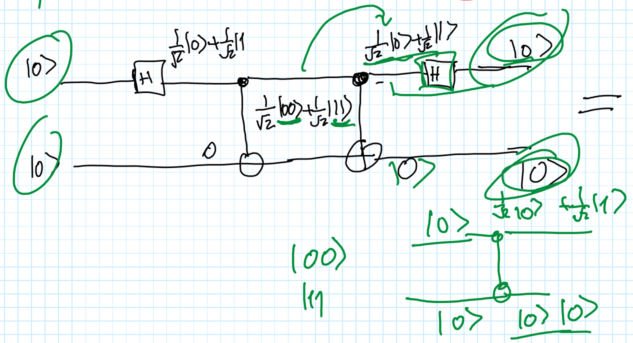
Bell Basis

$$\boxed{\langle \lambda_i | \lambda_j \rangle = \delta_{ij}}$$ orthogonal.

$\boxed{B^T B = I}$ B-orthogonal Matrices.

if $B^T = B$ $\quad \boxed{BB = I} \Longrightarrow B = B^{-1}$

$BB^{-1} = I$



$= I$

$\hat{H}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) =$

$= \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}|0\rangle +$

$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}|1\rangle =$

$= \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix} =$

$= \frac{1}{2}\begin{pmatrix}1\\ 1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\ -1\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix} = |0\rangle$

Superdense Coding

Charles Bennet 1992 Exp. 1996
Stephen Wiesner

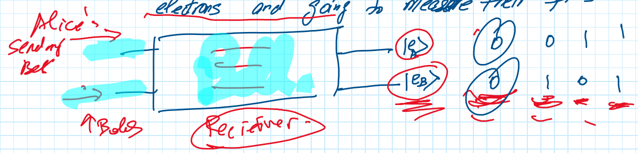
(1) Two electrons have the entangled spin state $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$ (#B, #B)

- One e⁻ goes to Alice — far apart
- Other e⁻ goes to Bob

(2) Alice wants to send Bob two classical bits of info
00, 01, 10 or 11

3) She is going to do this by sending electrons back to Bob after passing it through some gates circuits

4) Alice and Bob both have one electron each
- eventually Bob is going to have both electrons and going to measure their spins



5) Alice Pauli Gates

$I|0\rangle = |0\rangle$
$I|1\rangle = |1\rangle$

— [I] — 00

0 — [X] — 1
01

$\hat{x}|0\rangle = |1\rangle$
$\hat{x}|1\rangle = |0\rangle$

— [Z] — 10

$\hat{z}|0\rangle = |0\rangle$
$\hat{z}|1\rangle = -|1\rangle$

— [Y] — 11

$\hat{y}|0\rangle = -|1\rangle$
$\hat{y}|1\rangle = |0\rangle$

H and CNOT Are their own Inverse

6) Initially $\Rightarrow \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

$\Longrightarrow \frac{1}{\sqrt{2}}|0\rangle^{A} \otimes |0\rangle^{B} + \frac{1}{\sqrt{2}}|1\rangle^{A} \otimes |1\rangle^{B}$

7) She wants to send the information 00

$$I^{A}\left[\frac{1}{\sqrt{2}}|0\rangle \times |0\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |1\rangle\right] = \frac{1}{\sqrt{2}}|0\rangle \times |0\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |1\rangle$$

$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$$B^{-1}\left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\right)$$

$$B^{-1}|00\rangle = B^{-1}\left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\right) = |00\rangle$$

$$\boxed{|00\rangle} = \hat{B}^{-1}\left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\right) = |00\rangle \longrightarrow$$ Alice wanted to sent him

8) Alice wants to send Bob $\boxed{01}$?

$$\frac{1}{\sqrt{2}}|0\rangle^{A} \otimes |0\rangle^{B} + \frac{1}{\sqrt{2}}|1\rangle^{A} \otimes |1\rangle^{B}$$

$\hat{X}\left(\frac{1}{\sqrt{2}}|0\rangle\right) = \frac{1}{\sqrt{2}}|1\rangle$

$\hat{X}\left(\frac{1}{\sqrt{2}}|1\rangle\right) = \frac{1}{\sqrt{2}}|0\rangle$

$\hat{X}$ Gate

$\frac{1}{\sqrt{2}}|1\rangle \otimes |0\rangle + \frac{1}{\sqrt{2}}|0\rangle \otimes |1\rangle$

Bell $\hat{B}\left(\frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|01\rangle\right) = |01\rangle$

9) Alice wants to send 10

She will use $\hat{Z}$ gate.

$\hat{B}(\qquad) = |10\rangle$

10) Alice wants to send 11

She will use $\hat{\varphi}$ Gate.

$\hat{B}(\qquad) = |11\rangle$