QC16

Wednesday, March 31, 2021 3:56 PM

Quantum Teleportation

- Alice and Bob are far appart sharing entangled state $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$ (A B)
- Now Alice has another electron in the state
  extra $|\psi_e\rangle = a|0\rangle + b|1\rangle$ $a^2 + b^2 = 1$ $|\psi\rangle = \sum \alpha_n |n\rangle$
- Alice does not know values of a and b
- But Alice and Bob want to change Bobs qubit (e) that it become $a|0\rangle + b|1\rangle$
  (thus the qubit will appear at Bob's side)

$\Rightarrow$ To do this Alice needs to do

a) some manipulation with her initial and additional (e) $a|0\rangle + b|1\rangle$ qubit

b) and send Bob two classical bits (00 01 10 11) by conventional communication !

- Alice starts with $a|0\rangle + b|1\rangle$ and Bob ends getting it

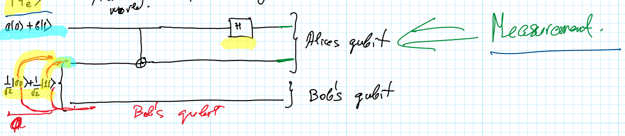
1) Bob and Alice share entangled state $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

2) Alice now gets extra electron in the state $a|0\rangle + b|1\rangle = |\psi_e\rangle$ which she wants to teleport to Bob

3) Idea is to entangle this e qubit (e) with her original qubit which is entangled with Bob's qubit (e).

4) we already know that CNOT Gate can do the entanglement

$|\psi_e\rangle$ Alice's wave.



5) start Alice and Bob

(a) $(a|0\rangle + b|1\rangle) \otimes \left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\right) = \frac{a}{\sqrt{2}}|000\rangle + \frac{a}{\sqrt{2}}|011\rangle + \frac{b}{\sqrt{2}}|100\rangle + \frac{b}{\sqrt{2}}|111\rangle = //$

$|\psi_e\rangle$

$\frac{a}{\sqrt{2}}\underbrace{|00\rangle}_{A} \otimes |0\rangle + \frac{a}{\sqrt{2}}\underbrace{|01\rangle}_{A} \otimes |1\rangle + \frac{b}{\sqrt{2}}\underbrace{|10\rangle}_{A} \otimes |0\rangle + \frac{b}{\sqrt{2}}|11\rangle \otimes |1\rangle$

6) Acting CNOT Gate to Alice's two e state

$\hat{CNOT}\left(\frac{a}{\sqrt{2}}|00\rangle \otimes |0\rangle + \frac{a}{\sqrt{2}}|01\rangle \otimes |1\rangle + \frac{b}{\sqrt{2}}|10\rangle \otimes |0\rangle + \frac{b}{\sqrt{2}}|11\rangle \otimes |1\rangle\right)$

$= \frac{a}{\sqrt{2}}\underbrace{|00\rangle}_{A} \otimes |0\rangle \quad \frac{a}{\sqrt{2}}\underbrace{|01\rangle}_{A} \otimes |1\rangle + \frac{b}{\sqrt{2}}\underbrace{|11\rangle}_{A} \otimes |0\rangle + \frac{b}{\sqrt{2}}\underbrace{|10\rangle}_{A} \otimes |1\rangle$

7) Now acting Hadamard gate to Alice's $|\psi_e\rangle$ — outpute. that came out from CNOT

$H\left(\frac{a}{\sqrt{2}}|00\rangle \otimes |0\rangle + \frac{a}{\sqrt{2}}|01\rangle \otimes |1\rangle + \frac{b}{\sqrt{2}}|11\rangle \otimes |0\rangle + \frac{b}{\sqrt{2}}|10\rangle \otimes |1\rangle\right)$

using $H|0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ $H|1\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

$\frac{a}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \otimes |0\rangle \otimes |0\rangle + \frac{a}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \otimes |1\rangle \otimes |1\rangle$

$+\frac{b}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle-\frac{1}{\sqrt{2}}|1\rangle\right)\otimes|1\rangle\otimes|0\rangle + \frac{b}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle-\frac{1}{\sqrt{2}}|1\rangle\right)\otimes|0\rangle\times|1\rangle =$

$|00\rangle$

$\frac{a}{2}|0\rangle\otimes|0\rangle\otimes|0\rangle + \frac{a}{2}|1\rangle|0\rangle|0\rangle + \frac{a}{2}|0\rangle|1\rangle|1\rangle$

$\frac{a}{2}|1\rangle|1\rangle|1\rangle + \frac{b}{2}|0\rangle|1\rangle|0\rangle - \frac{b}{2}|1\rangle|1\rangle|0\rangle$

$+\frac{b}{2}|0\rangle|0\rangle|1\rangle - \frac{b}{2}|1\rangle|0\rangle|1\rangle =$

Alice, Bob, $|\psi_e\rangle$, Alice, Bob

$\frac{1}{2}|00\rangle\otimes(a|0\rangle+b|1\rangle) + \frac{1}{2}|01\rangle(a|1\rangle+b|0\rangle)$

Alice, Bob, Alice, Bob.

$\frac{1}{2}|10\rangle\otimes(a|0\rangle-b|1\rangle) + \frac{1}{2}|11\rangle(a|1\rangle-b|0\rangle)$

– probability that Alice measures $|00\rangle \to \frac{1}{4}$

$|01\rangle \to \frac{1}{4}$

$|10\rangle \to \frac{1}{4}$

$|11\rangle \to \frac{1}{4}$

– She Measure $|00\rangle$ – $a|0\rangle+b|1\rangle = |\psi_e\rangle$

– She Measure $|01\rangle$ – $a|1\rangle+b|0\rangle$

– She Measure $|10\rangle$ – $a|0\rangle-b|1\rangle$

– She Measure $|11\rangle$ – $a|1\rangle-b|0\rangle$

Texting Bob

00

01

10

11

Bob

$\psi_e = a|0\rangle+b|1\rangle$

$\hat{X}[a|1\rangle+b|0\rangle] = a|0\rangle+b|1\rangle = |\psi_e\rangle$

$\hat{Z}[a|0\rangle-b|1\rangle] = a|0\rangle+b|1\rangle = |\psi_e\rangle$

$\hat{Y}[a|1\rangle-b|0\rangle] = a|0\rangle+b|1\rangle = |\psi_e\rangle$

⇒ Error Correction

digital 0 1 0 0 0 1 0 ...

00 11 000 000 00 11

⇒ Classical Error Correction (The Repetition Code)

– repeat any bit 3 times and send

0 – bit → 000 | Receiver | 001 → 000

1 bit → 111 | | 010 → 000

110 → 111

– The actual method is called $\boxed{\text{Parity test}}$

if bit c and d have same binary magnitude | Parity 0

c and d have different binary magnitude | Parity 1

– sign $\oplus$ $c\oplus d = \begin{cases} 0 & \text{same magn} \\ 1 & \text{dif magn} \end{cases}$

Exclusive OR

OR	$\oplus$		
T T	F	00	0
T F	T	10	1
F T	T	01	1
F F	F	11	0

– Suppose we recieved from the device 3 bits $b_0 b_1 b_2$

Knowing that — should be a repeated ... $b_0 = b_1 = b_2$

We check mutual parities

if $b_0 = b_1 = b_2 \Rightarrow b_0 \oplus b_1 = \boxed{0}$ and $b_0 \oplus b_2 = \boxed{0}$ → 000, 111

If $b_0 = b_1 \neq b_2$ then $b_0 \oplus b_1 = \boxed{0}$ and $b_0 \oplus b_2 = \boxed{1}$ (01, 10)

00(1) → 0
11(0) → 1

If $b_0 = b_2 \neq b_1 \Rightarrow b_0 \oplus b_1 = \boxed{1}$ and $b_0 \oplus b_2 = \boxed{0}$

if $b_0 \neq b_1 = b_2 \Rightarrow b_0 \oplus b_1 = \boxed{1}$ and $b_0 \oplus b_2 = \boxed{1}$ → 011, 100

- 00 — Nothing to correct
- 01 — $\boxed{\text{flip } b_2}$
- 10 — $\boxed{\text{flip } b_1}$
- 11 — $\boxed{\text{flip } b_0}$

— Can not do the same for Qubits, since we can not measure them

⇒ **Quantum Bit Flip Correction**

(1) Alice wants to send $\alpha|0\rangle + \beta|1\rangle$ to Bob

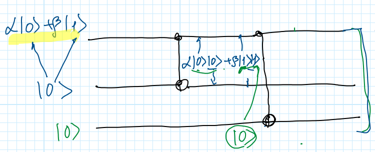
— We consider bit flipping error

$\alpha|1\rangle + \beta|0\rangle$! — cannot fan out

— but can fan out each pure qubits

(2) Alice does classical fan out to tripple each component $|0\rangle \longrightarrow |0\rangle|0\rangle|0\rangle$
$|1\rangle \longrightarrow |1\rangle|1\rangle|1\rangle$

This is done using CNOT



$\alpha|000\rangle + \beta|111\rangle$ — this is not cloned but different construction

(3) Alice sends $\alpha|000\rangle + \beta|111\rangle$ to Bob

qubits flipped due to noise

$$\begin{aligned} &\alpha|100\rangle + \beta|011\rangle \\ &\alpha|010\rangle + \beta|101\rangle \\ &\alpha|001\rangle + \beta|110\rangle \end{aligned}$$

! Consider this specific error scheme

(4) Bob wants detect the error and correct it

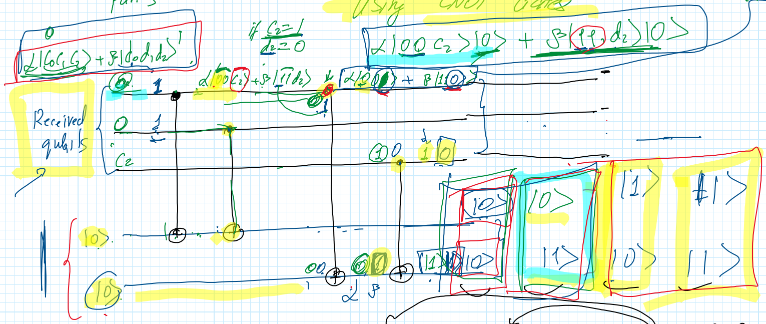
— He uses Parity Check Idea

— He adds an additional two qubits to

perform the parity check Using CNOT Gates

$\alpha|c_0c_1c_2\rangle + \beta|d_0d_1d_2\rangle$

$\alpha|00c_2\rangle|0\rangle + \beta|11d_2\rangle|0\rangle$



X

$(\alpha|001\rangle + \beta|110\rangle)$

$\alpha|00\rangle \underbrace{X|1\rangle}_{|0\rangle} + \beta|11\rangle \underbrace{X|0\rangle}_{|1\rangle} =$

$= \alpha|000\rangle + \beta|111\rangle$

	$c_0c_1c_2$	$d_0d_1d_2$
I	$\alpha|000\rangle$	$+\beta|111\rangle$
II	$\alpha|010\rangle$	$+\beta|101\rangle$
III	$\alpha|100\rangle$	$+\beta|011\rangle$

	I	II	III
$c_0 \oplus c_1$	0	1	1
$d_0 \oplus d_1$	0	1	1

$\boxed{c_0 \oplus c_1 = d_0 \oplus d_1} = 0$ / $= 1$

$b_0\ b_1\ b_2$

No errors

$|c_0c_1c_2\rangle = |000\rangle$

$|d_0d_1d_2\rangle = |111\rangle$

— Suppose Bob receives $\alpha|c_0c_1c_2\rangle + \beta|d_0d_1d_2\rangle$

First four qubits

(1) $\alpha|c_0c_1c_2\rangle + \beta|d_0d_1d_2\rangle|0\rangle = \alpha|c_0c_1c_2\rangle|0\rangle + \beta|d_0d_1d_2\rangle|0\rangle$

assume $\boxed{c_0 \oplus c_1 = d_0 \oplus d_1 = 0}$ — (1)

II $\alpha|c_0c_1c_2|0\rangle + \beta|d_0d_1d_2\rangle|0\rangle$

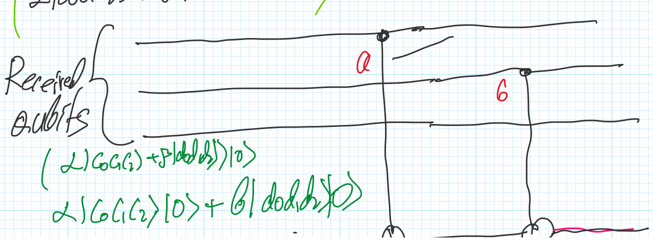
assume $\boxed{c_0 \oplus c_1 = d_0 \oplus d_1 = 1}$

$\Rightarrow$ Suppose Bob received $\alpha|c_0c_1c_2\rangle + \beta|d_0d_1d_2\rangle$

— Condition: If there is an error it happen in both $c_0c_1c_2$ and $d_0d_1d_2$ — exactly at the same place

1) Bob includes his four qubit $\boxed{c_0 \oplus c_1 = d_0 \oplus d_1}$ at the input Condition

$(\alpha|c_0c_1c_2\rangle + \beta|d_0d_1d_2\rangle)|0\rangle$



$(\alpha|c_0c_1c_2\rangle + \beta|d_0d_1d_2\rangle)|0\rangle$

$\alpha|c_0c_1c_2\rangle|0\rangle + \beta|d_0d_1d_2\rangle|0\rangle$

$|0\rangle$

I) if $c_0 \oplus c_1 = d_0 \oplus d_1 = 0$

(a) $a|c_0c_1c_2|0\rangle + b|d_0d_1d_2|0\rangle$

say $c_0=0$ $c_1=0$ $\qquad 1\,1$

Result $a|c_0c_1c_2|0\rangle + b|d_0d_1d_2|1\rangle$

(b) $a|c_0c_1c_2|0\rangle + b|d_0d_1d_2|0\rangle = (a|c_1c_2c_3\rangle + b|d_1d_2d_3\rangle)|0\rangle$

II) if $c_0 \oplus c_1 = d_0 \oplus d_1 = 1$

a) $a|c_0c_1c_2\rangle|0\rangle + b|d_0d_1d_2|0\rangle$

$\quad 1\,0 \qquad\qquad 0\,1$

$a|c_0c_1c_2|1\rangle + b|d_0d_1d_2|0\rangle$

$\quad 1\,0 \qquad\qquad 0\,1$

b) $(a\,c_0|c_1c_2|1\rangle + b|d_0d_1d_2|1\rangle) = (a|c_1c_2c_3\rangle + b|d_1d_2d_3\rangle)|1\rangle$

— In both cases '4' qubit is not entangled

$\Rightarrow$ Similar argument is applies to the fifth qubit

— It is $|0\rangle$ if $c_0 \oplus c_2 = d_0 \oplus d_2 = 0$

$|1\rangle$ $\quad c_0 \oplus c_2 = d_0 + d_2 = 1$

- Since 4 and 5 are not entangled with top three, Bob can make a measurement for 4 and 5

- if he gets 00 — then Nothing has to be done

- if he gets 01 — flips the third qubit using X Gate on 3rd wire

- if he gets 10 — X — on the second wire

- if he gets 11 — X — on the first wire