QC4

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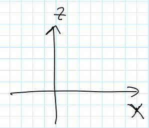
Entanglement)
(Mathematical Foundation)

- Consider Alice's and Bob's qubits at the same time

Alice $|\psi_A\rangle = \alpha_0|a_0\rangle + \alpha_1|a_1\rangle$

Bob $|\psi_B\rangle = \beta_0|b_0\rangle + \beta_1|b_1\rangle$

standard
$|a_0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ 0
$|a_1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ 1



- Consider their qubits together

$|\psi_{AB}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle =$

Keeping the order of $A \otimes B$ $\neq B \otimes A$ Please 0

$(\alpha_0|a_0\rangle + \alpha_1|a_1\rangle)(\beta_0|b_0\rangle + \beta_1|b_1\rangle) =$

$\underbrace{\alpha_0\beta_0}_{\gamma_{00}}\underbrace{|a_0\rangle\otimes|b_0\rangle}_{|\lambda_{00}\rangle} + \underbrace{\alpha_0\beta_1}_{\gamma_{01}}\underbrace{|a_0\rangle\otimes|b_1\rangle}_{|\lambda_{01}\rangle} + \underbrace{\alpha_1\beta_0}_{\gamma_{10}}\underbrace{|a_1\rangle\otimes|b_0\rangle}_{|\lambda_{10}\rangle} + \underbrace{\alpha_1\beta_1}_{\gamma_{11}}\underbrace{|a_1\rangle\otimes|b_1\rangle}_{|\lambda_{11}\rangle}$

$\boxed{\langle\lambda_{00}|\lambda_{00}\rangle = 1 \quad \langle a_0|a_0\rangle\ \langle b_0|b_0\rangle,\ \text{each} = 1}$

$\langle\lambda_{01}|\lambda_{10}\rangle$

$\langle\lambda_{00}|\lambda_{01}\rangle = 0$

$|\psi_{AB}\rangle = |\psi_A\rangle\otimes|\psi_B\rangle = \gamma_{00}|\lambda_{00}\rangle + \gamma_{01}|\lambda_{01}\rangle + \gamma_{10}|\lambda_{10}\rangle + \gamma_{11}|\lambda_{11}\rangle$

$|\psi\rangle = \sum_n \alpha_n|\lambda_n\rangle$

$\langle\psi_{AB}|\psi_{AB}\rangle = 1$

$\langle\psi|\psi\rangle = 1$

$\gamma_{00}^2 + \gamma_{01}^2 + \gamma_{10}^2 + \gamma_{11}^2 = 1$

$\sum_n |\alpha_n|^2 = 1$

$\Rightarrow$ New Orthonormal basis

$|\lambda_{00}\rangle \quad |\lambda_{01}\rangle, |\lambda_{10}\rangle, |\lambda_{11}\rangle$

$\langle\lambda_{ij}|\lambda_{mn}\rangle = \delta_{(ij)(mn)}$

- additional condition

$\gamma_{00}\cdot\gamma_{11} = \alpha_0\beta_0\alpha_1\beta_1$

equal. $\|$

$\gamma_{01}\cdot\gamma_{10} = \alpha_0\beta_1\alpha_1\beta_0$

$\rightarrow$ two $|\psi_A\rangle$ $|\psi_B\rangle$ states are $\boxed{\text{not entangled}}$

- if $\boxed{\gamma_{00}\gamma_{11} \neq \gamma_{01}\gamma_{10}}$ $\boxed{|\psi_A\rangle\ |\psi_B\rangle \text{ entangled}}$

$\boxed{\gamma_{00}+\gamma_{11}+\gamma_{01}+\gamma_{10}=1}$

$\Rightarrow$ we allow any possible value for $\gamma_{00}, \gamma_{11}, \gamma_{10}, \gamma_{11}$

only condition

$$\boxed{\gamma_{00}^2+\gamma_{01}^2+\gamma_{10}^2+\gamma_{11}^2=1}$$ entangled

$\Rightarrow$ Unentangled Qubits: Example.

$$|\psi_A\rangle = \frac{1}{\sqrt{2}}|a_0\rangle + \frac{1}{\sqrt{2}}|a_1\rangle$$

$$|\psi_B\rangle = \frac{1}{2}|b_0\rangle + \frac{\sqrt{3}}{2}|b_1\rangle$$

$$|\psi_{AB}\rangle = |\psi_A\rangle|\psi_B\rangle = \left(\frac{1}{\sqrt{2}}|a_0\rangle + \frac{1}{\sqrt{2}}|a_1\rangle\right)\left(\frac{1}{2}|b_0\rangle + \frac{\sqrt{3}}{2}|b_1\rangle\right)$$

Start from here

$$|\psi_{AB}\rangle = \frac{1}{2\sqrt{2}}|a_0\rangle|b_0\rangle + \underbrace{\frac{\sqrt{3}}{2\sqrt{2}}|a_0\rangle|b_1\rangle + \frac{1}{2\sqrt{2}}|a_1\rangle|b_0\rangle} + \frac{\sqrt{3}}{\sqrt{2}\,2}|a_1\rangle|b_1\rangle$$

outer $\boxed{\frac{\sqrt{3}}{8}}$ inner $\boxed{\frac{\sqrt{3}}{8}}$

$$\gamma_{00} = \frac{1}{2\sqrt{2}}, \quad \gamma_{01} = \frac{\sqrt{3}}{2\sqrt{2}}, \quad \gamma_{10} = \frac{1}{2\sqrt{2}} \quad \gamma_{11} = \frac{\sqrt{3}}{\sqrt{2}\cdot 2}$$

$$\gamma_{00}\cdot\gamma_{11} = \frac{\sqrt{3}}{4\cdot 2} \qquad \gamma_{01}\cdot\gamma_{10} = \frac{\sqrt{3}}{42}$$

not entangled

– Alice makes a measurement (Bob is not in the Picture)

$$|\psi_A\rangle = \frac{1}{\sqrt{2}}|a_0\rangle + \frac{1}{\sqrt{2}}|a_1\rangle$$

$$|\langle a_0|\psi_A\rangle|^2 = \boxed{\frac{1}{2}} \qquad |\langle a_1|\psi_A\rangle|^2 = \boxed{\frac{1}{2}}$$

0   1

– Alice makes a measurement (aware of Bob, and Bob did not make any measurements)

$$\langle a_0|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}\frac{1}{2}|b_0\rangle + \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2}|b_1\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{2}|b_0\rangle + \frac{\sqrt{3}}{2}|b_1\rangle\right)$$

Probability of measuring 0 (in $|a_0\rangle$ state)

$$|\langle a_0|\psi_{AB}\rangle|^2 = \frac{1}{2}\left(\frac{1}{4}+\frac{3}{4}\right) = \frac{1}{2}$$

$$\langle a_1|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{2}|b_0\rangle + \frac{\sqrt{3}}{2}|b_1\rangle\right)$$

1

Probability of measuring 1 (in $|a_1\rangle$ state)

$$|\langle a_1|\psi_{AB}\rangle|^2 = \boxed{\frac{1}{2}}$$

⇒ Alice's measurement did not change when Bob was in the picture and he did not do any measurement

⇒ Now consider situation when Alice makes a measurement after Bob did

– Assume Bob measured in his frame and got 0

this means Bob's system collapsed to $\boxed{|b_0\rangle}$

and

$$|\psi_{AB}\rangle = |\psi_A\rangle \otimes |b_0\rangle = \frac{1}{\sqrt{2}}|a_0\rangle|b_0\rangle + \frac{1}{\sqrt{2}}|a_1\rangle|b_1\rangle$$

$$\langle a_0|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}|b_0\rangle \qquad |\langle a_0|\psi_{AB}\rangle|^2 = \frac{1}{2}$$

$$\langle a_1|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}|b_1\rangle \qquad |\langle b_1|\psi_{AB}\rangle|^2 = \frac{1}{2}$$

- Assume Bob measured in his frame and got 1
  this means his qubit collapsed to $|b_1\rangle$

$$|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}|a_0\rangle|b_1\rangle + \frac{1}{\sqrt{2}}|a_1\rangle|b_1\rangle$$

$\Rightarrow$ Show that the Opposite is true too

$\Rightarrow$ if Alice makes measurement it does not change Bob's measurement Outcome

When the qubits are unentangled, a measurement of on of the qubits has no effect on the other has absolutely no effect on the other qubit

Unentangled Qubits are Factorizable