QC6

Wednesday, February 17, 2021 4:54 PM

Monday,February 17

Invariance of Entangled Qubits

- Start with unentangled Qubits $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}\otimes\begin{bmatrix}1\\0\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}$

- show that this are unentangled qubits

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}+\frac{1}{\sqrt{2}}\begin{bmatrix}0\\0\\1\\0\end{bmatrix}=\frac{1}{\sqrt{2}}\underset{A}{\begin{bmatrix}1\\0\end{bmatrix}}\otimes\underset{B}{\begin{bmatrix}1\\0\end{bmatrix}}+\frac{1}{\sqrt{2}}\underset{A}{\begin{bmatrix}0\\1\end{bmatrix}}\otimes\underset{B}{\begin{bmatrix}1\\0\end{bmatrix}}$$

- Make it Entangled with CNOT Gate

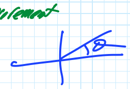
$$\widehat{CNOT}\begin{bmatrix}\frac{1}{\sqrt{2}}\\0\\\frac{1}{\sqrt{2}}\\0\end{bmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\0\\\frac{1}{\sqrt{2}}\\0\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$$

Show that this is Entangled:

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}+\frac{1}{\sqrt{2}}\begin{bmatrix}0\\0\\0\\1\end{bmatrix}=\frac{1}{\sqrt{2}}\underset{A}{\begin{bmatrix}1\\0\end{bmatrix}}\otimes\underset{B}{\begin{bmatrix}1\\0\end{bmatrix}}+\frac{1}{\sqrt{2}}\underset{A}{\begin{bmatrix}0\\1\end{bmatrix}}\otimes\underset{B}{\begin{bmatrix}0\\1\end{bmatrix}}$$

Prove that result is invariant with respect to the Choice of the Basis of the measurement

- Consider new orthonormal basis $(|x_0\rangle\ |x_1\rangle)$



Invariance means that

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\end{bmatrix}\otimes\begin{bmatrix}1\\0\end{bmatrix}+\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\end{bmatrix}\otimes\begin{bmatrix}0\\1\end{bmatrix}=\frac{1}{\sqrt{2}}|x_0\rangle\otimes|x_0\rangle+\frac{1}{\sqrt{2}}|x_1\rangle\otimes|x_1\rangle$$

① Start by writing $|x_0\rangle=\begin{pmatrix}a\\b\end{pmatrix}$ $|x_1\rangle=\begin{pmatrix}c\\d\end{pmatrix}$

② Next express our standard basis by Linear combination of $|x_0\rangle$ and $|x_1\rangle$

i.e

$\begin{pmatrix}1\\0\end{pmatrix}=\alpha_0|x_0\rangle+\alpha_1|x_1\rangle$

Remember the procedure

$$\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}=\begin{bmatrix}a\\c\end{bmatrix}\equiv\begin{pmatrix}\alpha_0\\\alpha_1\end{pmatrix}$$

Thus $\begin{pmatrix}1\\0\end{pmatrix}=a|x_0\rangle+c|x_1\rangle=a\begin{pmatrix}a\\b\end{pmatrix}+c\begin{pmatrix}c\\d\end{pmatrix}$

Using this

$$\begin{bmatrix}1\\0\end{bmatrix}\otimes\begin{bmatrix}1\\0\end{bmatrix}=\left(a\begin{bmatrix}a\\b\end{bmatrix}+c\begin{bmatrix}c\\d\end{bmatrix}\right)\otimes\begin{bmatrix}1\\0\end{bmatrix}=a\begin{bmatrix}a\\b\end{bmatrix}\otimes\begin{bmatrix}1\\0\end{bmatrix}+c\begin{bmatrix}c\\d\end{bmatrix}\otimes\begin{bmatrix}1\\0\end{bmatrix}$$

$$// = \begin{bmatrix} a \\ b \end{bmatrix} \otimes \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} \otimes \begin{bmatrix} c \\ d \end{bmatrix}$$

Thus $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \otimes \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} \otimes \begin{bmatrix} c \\ d \end{bmatrix}$

yes $\left(a\begin{bmatrix} a \\ b \end{bmatrix} + c\begin{bmatrix} c \\ d \end{bmatrix}\right) \otimes \left[a\begin{bmatrix} a \\ b \end{bmatrix} + c\begin{bmatrix} c \\ d \end{bmatrix}\right] = a^2\begin{bmatrix} a \\ b \end{bmatrix} \otimes \begin{bmatrix} a \\ b \end{bmatrix} + c^2\begin{bmatrix} c \\ d \end{bmatrix} \otimes \begin{bmatrix} c \\ d \end{bmatrix}$

$+ ac\begin{bmatrix} a \\ b \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix} + ca\begin{bmatrix} c \\ d \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix}$

Now we do it for $\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \beta_0|x_0\rangle + \beta_1|x_1\rangle$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} b \\ d \end{pmatrix} = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}$$

Thus $\begin{pmatrix} 0 \\ 1 \end{pmatrix} = b|x_0\rangle + d|x_1\rangle = b\begin{pmatrix} a \\ b \end{pmatrix} + d\begin{pmatrix} c \\ d \end{pmatrix}$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \left[b\begin{pmatrix} a \\ b \end{pmatrix} + d\begin{pmatrix} c \\ d \end{pmatrix}\right] \times \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} \times \begin{bmatrix} c \\ d \end{bmatrix}$$

yes: $\begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \left[b\begin{pmatrix} a \\ b \end{pmatrix} + d\begin{pmatrix} c \\ d \end{pmatrix}\right] \times \left[b\begin{pmatrix} a \\ b \end{pmatrix} + d\begin{pmatrix} c \\ d \end{pmatrix}\right]$

$$// = b^2\begin{bmatrix} a \\ b \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix} + d^2\begin{bmatrix} c \\ d \end{bmatrix} \times \begin{bmatrix} c \\ d \end{bmatrix} + bd\begin{pmatrix} a \\ b \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix} + db\begin{bmatrix} c \\ d \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

3) Summing $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} a \\ b \end{bmatrix} \otimes \begin{bmatrix} a \\ b \end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix} c \\ d \end{bmatrix} \otimes \begin{bmatrix} c \\ d \end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix} a \\ b \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix} c \\ d \end{bmatrix} \otimes \begin{bmatrix} c \\ d \end{bmatrix} = //$

$\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$// = \frac{1}{\sqrt{2}}\begin{bmatrix} a \\ b \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix} c \\ d \end{bmatrix} \otimes \begin{bmatrix} c \\ d \end{bmatrix} = \frac{1}{\sqrt{2}}|x_0\rangle \times |x_0\rangle + \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}|x_1\rangle \times |x_1\rangle$$

yes $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\end{bmatrix}\times\begin{bmatrix}1\\0\end{bmatrix}+\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\end{bmatrix}*\begin{bmatrix}0\\1\end{bmatrix}$

$$=\frac{1}{\sqrt{2}}\Big(a^2\begin{bmatrix}a\\b\end{bmatrix}\times\begin{bmatrix}a\\b\end{bmatrix}+c^2\begin{bmatrix}c\\d\end{bmatrix}\begin{bmatrix}c\\d\end{bmatrix}+ac\begin{bmatrix}a\\b\end{bmatrix}\begin{bmatrix}c\\d\end{bmatrix}+ca\begin{bmatrix}c\\d\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}$$

$$b^2\begin{bmatrix}a\\b\end{bmatrix}\times\begin{bmatrix}a\\b\end{bmatrix}+d^2\begin{bmatrix}c\\d\end{bmatrix}\begin{bmatrix}c\\d\end{bmatrix}+bd\begin{bmatrix}a\\b\end{bmatrix}\begin{bmatrix}c\\d\end{bmatrix}+db\begin{bmatrix}c\\d\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}\Big)=$$

$$=\frac{1}{\sqrt{2}}\Big((a^2+b^2)\begin{bmatrix}a\\b\end{bmatrix}\times\begin{bmatrix}a\\b\end{bmatrix}+(c^2+d^2)\begin{bmatrix}c\\d\end{bmatrix}\begin{bmatrix}c\\d\end{bmatrix}+[ac+bd]\begin{bmatrix}a\\b\end{bmatrix}\begin{bmatrix}c\\d\end{bmatrix}$$
$$+[ca+db]\begin{bmatrix}c\\d\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}\Big)$$

But $a^2+b^2=1$ since $\alpha_0^2+\alpha_1^2=1$

$c^2+d^2=1$ since $\beta_0^2+\beta_1^2=1$

$ac+bd=0$ since $\langle\alpha_0|\alpha_1\rangle=0=(a\ b)\begin{pmatrix}c\\d\end{pmatrix}=ac+bd=0$

$ca+db=0$

One obtains

$$\begin{bmatrix}1\\0\end{bmatrix}\times\begin{bmatrix}1\\0\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix}a\\b\end{bmatrix}\times\begin{bmatrix}a\\b\end{bmatrix}+\frac{1}{\sqrt{2}}\begin{bmatrix}c\\d\end{bmatrix}\begin{bmatrix}c\\d\end{bmatrix}=\frac{1}{\sqrt{2}}|\alpha_0\rangle\times|\alpha_0\rangle+\frac{1}{\sqrt{2}}|\alpha_1\rangle\times|\alpha_1\rangle$$