Take Home Final Exam

1. (20 points) Calculate eigenstates of $\hat{L}_z$ operator. Demonstrate that eigenvaleus of this operator are integer numbers.

2. (40 points) Calculate eigenstates of $\hat{L}^2$ operator. Present a detailed derivation.

3. (20 points) Calculate $\hat{L}_x \psi_{l,m}$ and $\hat{L}_y \psi_{l,m}$ where $\psi_{l,m}$ eigenstates of $\hat{L}^2$ and $\hat{L}_z$ operators

4. (20 points) Using the *expression* :

$$Y_l^m(\theta, \phi) = (-1)^{\frac{m+|m|}{2}} \left[\frac{2l+1}{4\pi} \left(\frac{(l-[m])!}{(l-[m])!}\right)\right]^{\frac{1}{2}} P_l^{|m|}(\cos\theta)\, e^{im\phi}$$

where $P_l^{|m|}(\cos\theta)$ is Associated Legendre Function.

Show that $Y_l^m(\pi-\theta, \phi+\pi) = (-1)^l Y_l^m(\theta, \phi)$.

Explain why this relation is related to the parity of the quantum state