Homework 4 (10 points each problem)

1. Show how the following transformation law is obtained

$$A'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} A^{\nu}.,$$

Then usin the above and the definition of scalar product obtain the transformation relation for A_{μ}'.

2. Obtain the transformation relation for $g_{\mu\upsilon}$ and $g^{\mu\upsilon}$ as well as calculate $g_{\mu\alpha}\, g^{\beta\upsilon}$.

3. Show that the tensor relations are invariant with respect to the general transformation (covariance theorem).

4. Show that Affine connection is not a true tensor.

5. Using relation $\Gamma^{\lambda}_{\mu\nu} = \left\{ {\lambda \atop \mu\nu} \right\}$ prove that $\Gamma_{\kappa\mu\nu} + \Gamma_{\mu\kappa\nu} = g_{\mu\kappa,\upsilon}$

6. Show that $\frac{dA^{\mu}}{dx^{\nu}}$ is not a true tensor, while the covariant derivative $A^{\mu}{}_{;\nu} = \frac{dA^{\mu}}{dx^{\nu}} + \Gamma^{\mu}{}_{\sigma\nu} A^{\sigma}$ is a true tensor.

7. From $A^{\mu}{}_{;\nu} = \frac{dA^{\mu}}{dx^{\nu}} + \Gamma^{\mu}{}_{\sigma\nu} A^{\sigma}$ obtain the covariant derivative for second rank contravariant tensor : $T^{\mu\nu}$

8. Show that $g_{\mu\nu;\lambda} = g^{\mu\nu}{}_{;\lambda} = 0$