

Homework 8

1. (20 points) The Friedman - Lamaitre - Roberson - Walker metric is given as follows

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{a[t]^2}{1-kr^2} & 0 & 0 \\ 0 & 0 & -a[t]^2 r^2 & 0 \\ 0 & 0 & 0 & -a[t]^2 r^2 (\sin[\theta])^2 \end{pmatrix}$$

calculate Affine connections

$$\Gamma_{11}^0 = \frac{a(t) a'(t)}{c(1-kr^2)}, \Gamma_{22}^0 = \frac{r^2 a(t) a'(t)}{c}, \Gamma_{33}^0 = \frac{r^2 a[t] \sin[\theta]^2 a'[t]}{c},$$

$$\Gamma_{11}^1 = \frac{kr}{1-kr^2}, \Gamma_{22}^1 = -r(1-kr^2), \Gamma_{33}^1 = -r(1-kr^2) \sin[\theta]^2,$$

$$\Gamma_{01}^1 = \frac{a'(t)}{ca(t)}, \Gamma_{33}^2 = -\sin(\theta) \cos(\theta), \Gamma_{02}^2 = \frac{a'[t]}{ca[t]}, \Gamma_{12}^2 = \frac{1}{r},$$

$$\Gamma_{03}^3 = \frac{a'(t)}{ca(t)}, \Gamma_{13}^3 = \frac{1}{r},$$

$$\Gamma_{23}^3 = \cot(\theta) \text{ and all other terms are } 0.$$

2. (20 points) Using above results calculate the Ricci tensor and Scalar Curvature :

$$R_{00} = \frac{3 a''[t]}{c^2 a[t]},$$

$$R_{11} = \frac{2 c^2 k + 2 a'[t]^2 + a[t] a''[t]}{c^2 (-1 + k r^2)},$$

$$R_{22} = -\frac{r^2 (2 c^2 k + 2 a'[t]^2 + a[t] a''[t])}{c^2},$$

$$R_{33} = R_{22} \sin[\theta]^2, \text{ and all other terms are } 0.$$

For Scalar curvature obtain :

$$R = \frac{6 (c^2 k + a'[t]^2 + a[t] a''[t])}{c^2 a[t]^2}$$

3. (20 points) Derive Einstein Tensor in mixed representation using

Ricci tensors and scalar curvature :

$$G_0^0 = - \frac{3 (c^2 k + a' [t]^2)}{c^2 a [t]^2}$$

$$G_1^1 = G_2^2 = G_3^3 = - \frac{c^2 k + a' [t]^2 + 2 a [t] a'' [t]}{c^2 a [t]^2}$$

4. (40 points) Derive Cosmological Equations :

$$(1) \left(\frac{a' [t]}{a [t]} \right)^2 + \frac{k c^2}{a [t]^2} - \frac{\Lambda c^2}{3} = \frac{8 \pi G}{3} \rho$$

$$(2) \frac{2 a'' [t]}{a [t]} + \left(\frac{a' [t]}{a [t]} \right)^2 + \frac{k c^2}{a [t]^2} - \Lambda c^2 = - \frac{8 \pi G}{c^2} P$$

5. (20 points) From the above obtain cosmological equation for the time

dependence of the density. Solve it for the case of the

Equation of State of

Perfect Liquid.

6. (20 points) Choosing Eq. (1) from Problem 4 and the equation from Problem 5.

solve the Cosmological equations for Matter dominated universe for the

case of zero Cosmological constant and curvature $k = 0, 1, -1$.

Graph the time dependence of the scale factor for above three cases.