John Von Neumann and Cellular Automata in Physics

Sandip Baral

Florida International University

Dec 1, 2025

outline

- John von Neumann (1903–1957) was one of the mathematician/physicist of the twentieth century, contributing to quantum mechanics, operator theory, numerical hydrodynamics, and computer architecture.
- The early development of *cellular automata*—a discrete, rule-based modeling approach capable of producing complex behavior from simple interactions.

Neumann's investigated discrete dynamical systems on a grid where each cell has a finite set of states.

First formal defination of a CA:

- a regular lattice of cells,
- a finite number of states per cell,
- a neighborhood scheme,
- and a determinstic update rule

1. 29-State Self Reproducing Automaton
2. Impact on Physics

Mathematical Structure of CA

A cellular automaton is defined on a lattice L with discrete time steps $t = 0, 1, 2, \ldots$. Each cell has a state $s_i(t)$ drawn from a finite set S. The time evolution follows a local rule:

$$s_i(t+1) = f\big(s_i(t), s_{j \in N(i)}(t)\big), \tag{1}$$

where $N(i)$ is the neighborhood of cell i. Common neighborhood schemes include:

- **von Neumann neighborhood:** 4 orthogonal neighbors.
- **Moore neighborhood:** 8 surrounding neighbors.

Types of CA

- Elementary CA
- Totalistic CA
- Probabilistic CA
- Reversible CA
- Lattice Gas Automata(LGA)
- Lattice Boltzmann Cellular Automata

Application in Physics

1. Statistical Mechanics and Critical Phenomena (including ising model dynamics, spin-lattice interaction)
2. Nonlinear Dynamics and Pattern Formation (including wave propagation, crystal growth)
3. Cellular automata for Fluid Dynamics (including LGA, LBM)
4. Quantum Cellular Automata (including quantum computation, Simulations of Dirac and KG equations.)

Lattice Gas Automata

$\rho(\frac{\partial u}{\partial t} + u.\nabla u) = -\nabla p + \mu\nabla^2 u$

,where $\rho =$ fluid density, u = fluid velocity vector, $\frac{\partial u}{\partial t}=$ local acceleration, $u.\nabla u =$ convective acceleration, $-\nabla p=$ pressure gradient force, $\mu\nabla^2 u =$ pressure accelerate from hp to lp.

lattice Gas Automata

- A rectangular 2D grid filled with small colored cells.
- The colors continuously change as the particles move and collide.
- The motion appears random but follows strict physical rules.
- We see patterns forming, dissolving, and moving across the grid.
- This collision rules conserves mass and momentum

Lattice Boltzmann Method An origin of LGA, the Lattice Boltzmann Method (LBM) uses a discrete velocity distribution function that can be expressed as:

$$f_i(\mathbf{x} + \mathbf{c}_i \Delta t, t + \Delta t) - f_i(\mathbf{x}, t) = \Omega_i(f), \tag{2}$$

which recovers macroscopic hydrodynamics through the Chapman–Enskog expansion.This expansion shows viscosity is related to relaxation time.

Von Neumann's Vision and Modern Physics

Neumann thought that physical laws may fundamentally be computational. CA illustrate:

- **Emergence:** complex macroscopic behavior from simple microscopic rules.
- **Self-organization:** spontaneous formation of order.
- **Discrete models of nature:** physics can arise from discrete computational processes.
- **Universality:** CA can perform any computation, analogous to universal physical laws.

His ideas prognosticated modern perspectives on complexity, computational universes, and information-based physics.

Conclusion

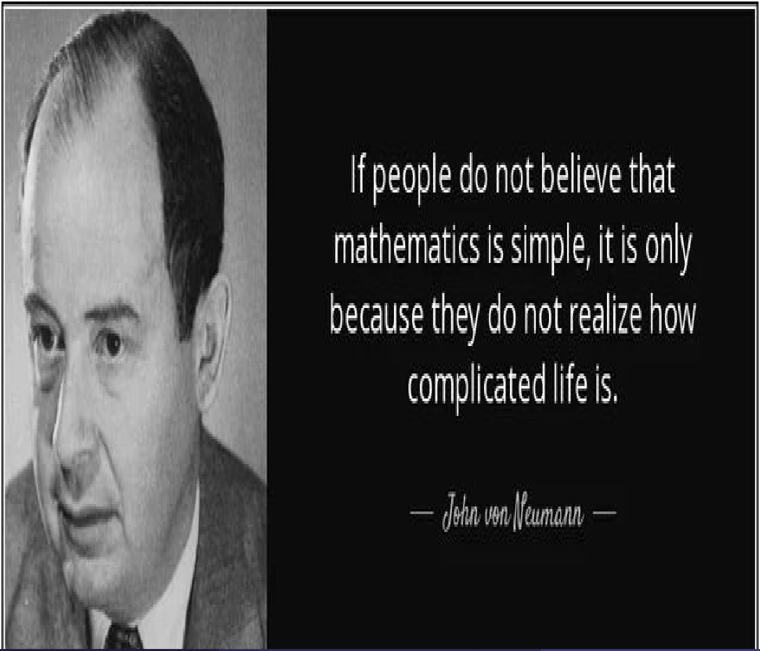
- John von Neumann's groundbreaking work on cellular automata laid the foundation for significant enhancements in computational physics, nonlinear dynamics, and complex systems.
- CA provide a potent architecture for modeling emergent behaviors, fluid dynamics, and phase transitions through discrete, simple local interactions.
- Neumann's insights continue to shape modern physics, where simple rules and local interactions are perceived as central to the emergence of complex physical phenomena.

References

- J. von Neumann, *Theory of Self-Reproducing Automata*, edited by A. W. Burks (University of Illinois Press, 1966).
- S. Ulam, "Random processes and transformations," *Proc. Int. Cong. Math.*, 2, 264 (1952).
- S. Wolfram, *A New Kind of Science*, Wolfram Media (2002).
- U. Frisch, B. Hasslacher, and Y. Pomeau, "Lattice-gas automata for the Navier–Stokes equation," *Phys. Rev. Lett.* 56, 1505 (1986).
- S. Succi, *The Lattice Boltzmann Equation*, Oxford University Press (2001).

1. Prof. Dr. Misak M. Sargsian
2. My Colleagues
3. My family

Thank you!


If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.
— John von Neumann —