EMMY NOETHER AND THE SYMMETRIES OF NATURE

A look at Noether’s Theorem and its impact on physics.

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WHO WAS EMMY NOETHER?

- Emmy Noether (1882–1935) was a German mathematician who transformed algebra and theoretical physics.
- She worked at a time when women were not allowed to officially teach; she often lectured under David Hilbert's name.
- Despite this, she became known for extraordinary creativity in abstract algebra and mathematical physics.
- Einstein praised her, saying: "She was the most significant creative mathematical genius thus far produced since the higher education of women began."

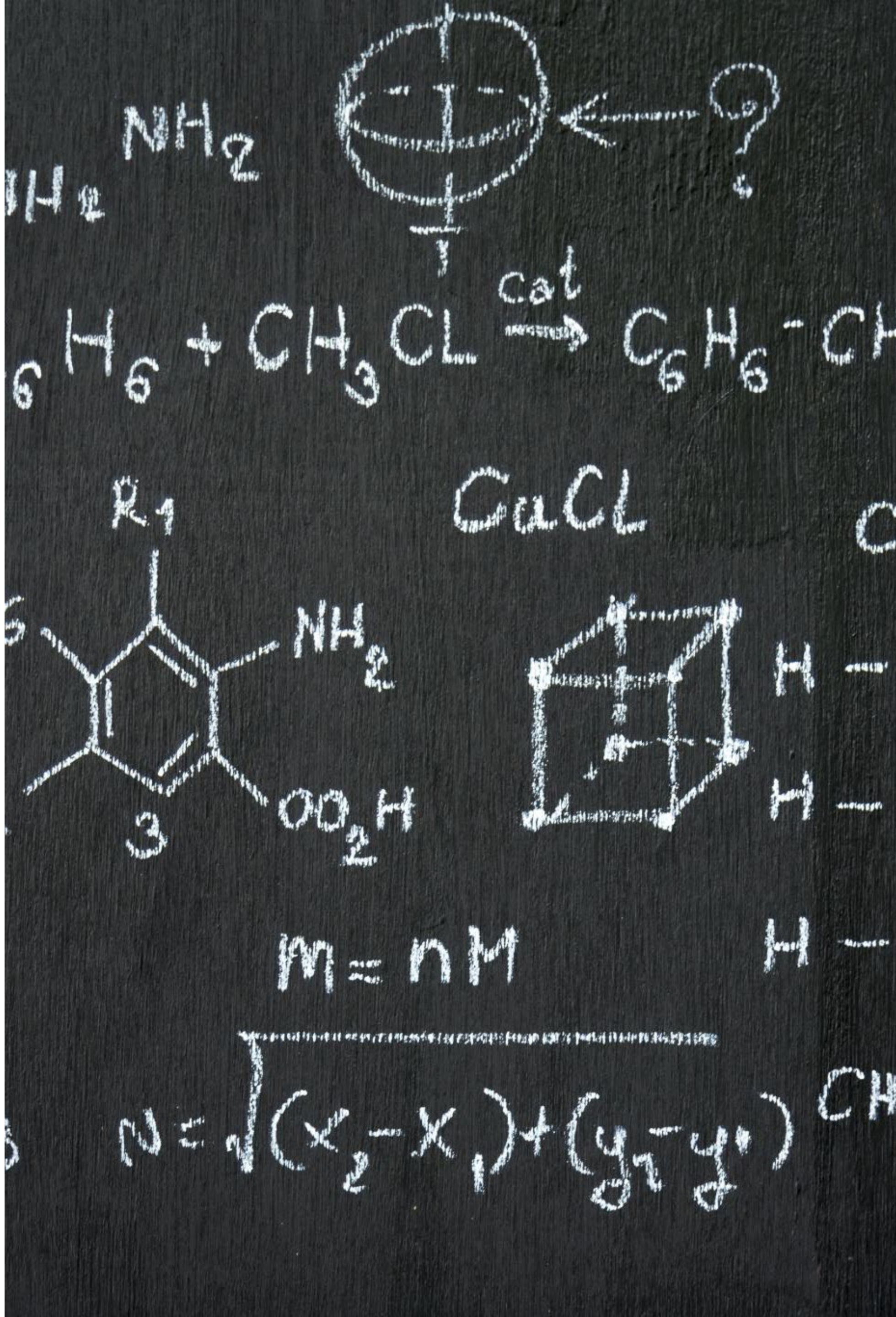
WHY NOETHER'S WORK WAS NEEDED

Before Noether, conservation laws were observed but unexplained.

- Physics had long known conservation laws:
  - Energy is conserved
  - Momentum is conserved
  - Angular momentum is conserved
  - Electric charge is conserved
- But no one knew why.
- These were given facts with no deeper explanation.
- Physics lacked a unifying principle explaining why nature behaves so consistently.

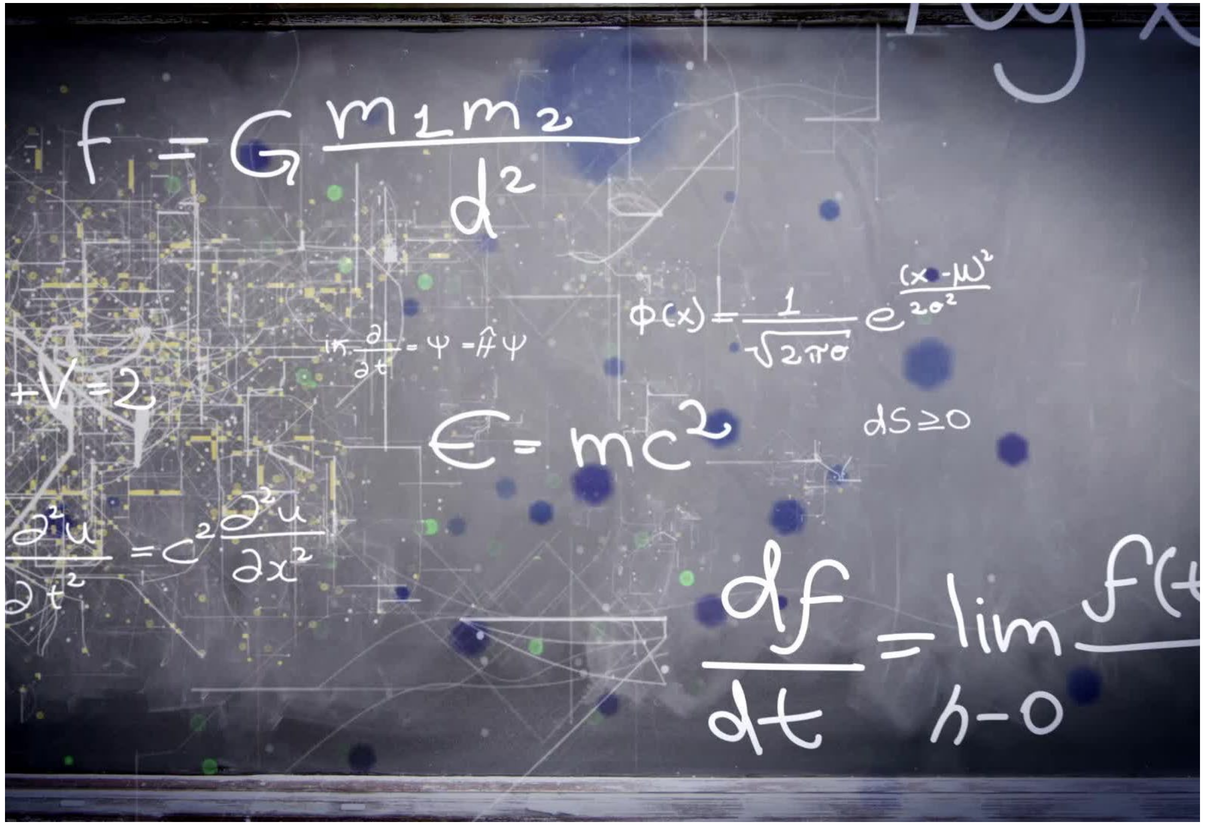
WHAT IS A SYMMETRY IN PHYSICS?

- A symmetry is a transformation that does not change the form of the laws of physics.
- Symmetries are not about objects looking the same, they're about the laws of nature behaving the same.
- Examples of transformations:
  - Shifting time forward
  - Moving something left or right
  - Rotating a system around an axis
  - Changing quantum phases
- If the equations remain unchanged under one of these transformations, nature has a symmetry.



EXAMPLES OF SYMMETRIES IN DAILY LIFE

- These aren't physical symmetries, but they help visualize the concept at hand:
  - A perfect circle looks the same if you rotate it → rotational symmetry.
  - A brick wall pattern repeats → translational symmetry.
  - A snowflake has 6-fold symmetry — rotations of 60° preserve its shape.
- Now imagine this idea, but for equations, not objects.



NOETHER'S THEOREM (CONCEPTUAL)

- "Every continuous symmetry of the laws of physics corresponds to a conserved quantity."
- This means:
  - If time doesn't matter → energy must be conserved.
  - If position doesn't matter → momentum must be conserved.
  - If direction doesn't matter → angular momentum must be conserved.
  - If quantum phase doesn't matter → electric charge is conserved.
- Noether turned conservation laws from observations into consequences of symmetry.

TIME SYMMETRY → ENERGY CONSERVATION

- Symmetry:
  - The laws of physics do not change over time.
  - Gravity works the same today as it did yesterday.
- Conserved Quantity:
  - Energy.
- Examples:
  - A pendulum slows and speeds up, but total energy stays constant.
  - Planetary orbits keep the same total energy over billions of years.
  - Nuclear reactions conserve total energy, even if mass becomes energy.
- If physics changed day to day, energy conservation would break.

SPATIAL SYMMETRY → MOMENTUM CONSERVATION

Symmetry:

- The laws of physics are the same everywhere in space.
- Dropping a ball in New York obeys the same laws as dropping it in Tokyo.

Conserved Quantity:

- Momentum.

Examples:

- A rifle pushes backward as a bullet shoots forward, equal and opposite momentum.
- Rockets in space move by ejecting gas backward.
- When two ice skaters push off each other, they glide in opposite directions with conserved momentum.

If the universe had "special spots" where physics changed, momentum would not be conserved.

Symmetry:

- Rotating the system does not change the laws of physics.

Conserved Quantity:

- Angular momentum.

Examples:

- A figure skater spins faster when pulling in their arms, angular momentum is conserved.
- Earth keeps spinning because nothing absorbs its angular momentum.
- Tornadoes and hurricanes spin due to conservation of angular momentum.

This symmetry explains orbits, gyroscopes, spinning tops, basically everything that rotates.

ROTATIONAL SYMMETRY ⟶ ANGULAR MOMENTUM

NOETHER'S THEOREM MATHEMATICALLY

- If the action

$$S[q] = \int L(q, \dot{q}, t)\, dt$$

- is invariant under a continuous transformation,

$$q \;\rightarrow\; q + \epsilon$$

- then the Noether quantity

$$J = \frac{\partial L}{\partial \dot{q}}\, \delta q$$

- is conserved.

$$\frac{dJ}{dt} = 0.$$

- Where delta q is the infinitesimal change in the coordinate produced by the symmetry.

$$\delta q = \frac{d}{d\epsilon}(q + \epsilon)\Big|_{\epsilon=0}$$

NOETHER'S THEOREM MATHEMATICALLY (EXAMPLE)

- Starting from the Lagrange with no potential:

$$L = \frac{1}{2} m\dot{q}^2$$

- We apply a spatial transformation:

$$q \to q + \epsilon$$

- Since L has no q-dependance then:

$$\delta q = \frac{d}{d\epsilon}(q+\epsilon)\Big|_{\epsilon=0} = 1$$

- So, the Noether quantity is:

$$J = \frac{\partial L}{\partial \dot{q}}\delta q = m\dot{q}\cdot 1 = p$$

- So, it is shown that translational symmetry gives rise to conservation of momentum:

$$\frac{d}{dt}(m\dot{q}) = 0,$$

$$p = m\dot{q} = constant.$$

IMPACT ON GENERAL RELATIVITY

Why GR caused a problem:

- In Newtonian physics, energy conservation is simple: kinetic + potential stays constant.
- But in General Relativity, gravity is not a force, it is the curvature of spacetime.
- Because spacetime curves differently in different regions, there is no single, global way to define "total energy."

What confused Einstein:

- He found that energy seemed not to be conserved in GR.
- For example, gravitational waves carry energy, but where does it come from?
- Einstein realized something was wrong with the old way of thinking about conservation.

Noether's solution:

- Noether showed that conservation laws depend on symmetries of spacetime itself.
- If spacetime has time-translation symmetry → energy is conserved.
- If spacetime has spatial-translation symmetry → momentum is conserved.
- In curved spacetime, these symmetries are local, not global.
- This explained why global energy conservation is not always possible in GR, spacetime may not have the required symmetry.

Concrete example:

- The expanding universe (cosmology) has no global time symmetry.
- Therefore: energy is not globally conserved in an expanding universe.
- This is why the energy of photons decreases (cosmological redshift) as the universe expands.
- Before Noether, this seemed paradoxical, now it's mathematically understood.

SYMMETRY BREAKING

The big idea: Symmetry breaking means the laws have a symmetry, but the state of the system does not.

1. Example: The Mexican Hat Potential

- Imagine a perfectly symmetric “donut-shaped” potential (a circular valley).
- The system is symmetric under any rotation.
- But the particle must choose one point in the valley to sit.
- The laws are symmetric → the chosen state is not.

2. Physical Example: Magnetism (Spontaneous Symmetry Breaking)

- At high temperature, atoms are randomly oriented → rotational symmetry.
- When cooled below the Curie temperature, atoms all align in one direction.
- Nature “chooses a direction.”
- The laws remain rotationally symmetric, but the magnet is not.
- This is a real macroscopic example of symmetry breaking.

Why Symmetry Breaking Matters

- It explains structures in matter, chemistry, stars, and galaxies.
- It shapes the evolution of the universe.
- Symmetry breaking is as important as symmetry itself.

LEGACY OF EMMY NOETHER

- Revolutionized physics through symmetry.
- Unified all conservation laws under one idea.
- Modern physics relies heavily on symmetry principles.
- She is considered one of the greatest mathematicians of the 20th century.
- Inspired generations of mathematicians and physicists.
- Forms the backbone of classical mechanics, quantum mechanics, GR, and particle physics.
- Noether didn’t just solve a problem; she changed the foundation of physics.

SUMMARY

- Symmetry → Conservation laws.
- Noether's theorem explains why the universe behaves consistently.
- Every modern physical theory uses her ideas.
- She is one of the greatest mathematicians in history.

QUESTIONS?

Thank you!