Possible Exam Questions

1. Show that harmonic series diverge diverge logarithmically

2. Prove that $\zeta(2) = \sum_{n=1}^{\infty} n^{-2}$ is converging

3. Prove the Leibniz criterion of convergence for alternating series

4. Obtain Taylor series for $\ln(1+x)$ function and find the range of convergence

5. Prove the uniqueness theorem for power series.

6. Express $\sin(x)$ and $\cos(x)$ functions through the power series.

7. Using Levi – Civita representation of cross product prove (a) $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

(b) $\vec{A}\cdot\left(\vec{B}\times\vec{C}\right) = \vec{B}\cdot\left(\vec{C}\times\vec{A}\right) = \vec{C}\cdot\left(\vec{A}\times\vec{B}\right)$

(c) $\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \vec{B}\left(\vec{A}\cdot\vec{C}\right) - \vec{C}\left(\vec{A}\cdot\vec{B}\right)$

8. Show *that* :

$$\left(\vec{A}\times\vec{B}\right)\cdot\left(\vec{C}\times\vec{D}\right) = \left(\vec{A}\cdot\vec{C}\right)\left(\vec{B}\cdot\vec{D}\right) - \left(\vec{A}\cdot\vec{D}\right)\left(\vec{B}\cdot\vec{C}\right)$$

and

$$\left(\vec{A}\times\vec{B}\right)\times\left(\vec{C}\times\vec{D}\right) = \left(\vec{A}\cdot\vec{B}\times\vec{D}\right)\vec{C} - \left(\vec{A}\cdot\vec{B}\times\vec{C}\right)\vec{D}$$

9. Show that scalar product $\vec{A}\cdot\vec{B} = |A|\,|B|\cos(\vartheta)$ and $\left|\vec{A}\times\vec{B}\right| = |A|\,|B|\sin(\vartheta)$ where ϑ is the relative angle.

10. Calculate the gradient of $f(r)\, r^n$ and consider cases of $f = 1$,
$n = 1$,
and $f = 1$ and $n = -1$.

11. Calculate divergence of $f(r)\, r^n\, \hat{r}$ ases of $f = 1$, $n = 1$,
and $f = 1$ and $n = -2$.

12. Calculate curl of $f(r)\, r^n\, \hat{r}$.

13. Calculate curl of $-z\, \hat{e}_x + x\, \hat{e}_y$

14. Calculate $\vec{\nabla} \cdot \vec{\nabla}\, \phi$ and then consider case of $\phi = r^n$

15. Calculate $\vec{\nabla} \times \vec{\nabla}\, \phi$ and $\vec{\nabla} \cdot \left(\vec{\nabla} \times \vec{V}\right)$ and $\vec{\nabla} \times \left(\vec{\nabla} \times \vec{V}\right)$

16. Calculate $\vec{\nabla} \cdot \left(f(r)\, \vec{V}\right)$ and $\vec{\nabla} \times \left(f(r)\, \vec{V}\right)$

17. Show that $\vec{\nabla}\left(\vec{A} \cdot \vec{B}\right) = \left(\vec{B} \cdot \vec{\nabla}\right) \vec{A} + \left(\vec{A} \cdot \vec{\nabla}\right) \vec{B} + \vec{A} \times \left(\vec{\nabla} \times \vec{B}\right) + \vec{B} \times \left(\vec{\nabla} \times \vec{A}\right)$

18. Show that

$$\int \vec{A}(r) \cdot \vec{\nabla} f(r)\, d^3 r = -\int f(r) \left(\vec{\nabla} \cdot \vec{A}(r)\right) d^3 r$$

$$\int \vec{V}(r) \cdot \left(\vec{\nabla} \times \vec{A}(r)\right) d^3 r = \int \vec{A}(r) \cdot \left(\vec{\nabla} \times \vec{V}(r)\right) d^3 r$$

19. Show that

$$\oint_{\delta V} F(r)\, d\vec{\sigma} = \int_V \vec{\nabla} F(r)\, d^3 r$$

$$\oint_{\delta V} d\vec{\sigma} \times \vec{V}(r) = \int_V \vec{\nabla} \times \vec{V}(r)\, d^3 r$$

20. Show that

$$\int_S d\vec{\sigma} \times \vec{\nabla} F(r) = \oint_{\delta S} F(r)\, d\vec{r}$$

$$\int_S \left(d\vec{\sigma} \times \vec{\nabla}\right) \times \vec{V}(r) = \oint_{\delta S} d\vec{r} \times \vec{V}(r)$$

21. Express Maxwell equations through scalar and vector potentials using Lorentz gauge

22. Derive Gauss ' Law for point charge and for continuous charge distribution

23. Show that $\nabla^2 \left(\frac{1}{r}\right) = -4\pi\delta(r)$

24. Calculate the following functions

$\sin^{-1}(z)$, $\sinh^{-1}(z)$

$\tan^{-1}(z)$, $\tanh^{-1}(z)$

$\cos^{-1}(z)$, $\cos^{-1}(z)$

25. Prove that

$$\sum_{n=0}^{N-1} \cos(nx) = \frac{\sin(Nx/2)}{\sin(x/2)} \cos(N-1)\frac{x}{2}$$

26. Find the analytic function

$w(z) = u(x, y) + i\, v(x, y)$

if

(a) $u(x, y) = x^3 - 3xy^2$

(b) $u(x, y) = e^{-y} \cos(x)$

27. For following $f(z)$ functions calculate $f'(z)$ and identify the maximal region within which $f(z)$ is analytic

(a) $f(z) = \frac{\sin(z)}{z}$

(b) $f(z) = \frac{1}{z(z+1)}$

(c) $f(z) = \tan(z)$

(d) $f(z) = e^{-1/z}$

28. Show that for analytic functions, the real and imaginary parts of the function satisfy Laplace Equation

29. Calculate $\oint_C z^n \, dz$ for integers $n \geq -1$

30. Prove Cauchy's Integral Theorem

31. Calculate

$\oint_C \frac{dz}{z^2 + z}$ for circle C defined by $|z| = R > 1$

32. Show that

$\oint_C z^{m-n-1} \, dz$ were m and n are integers is

Kronecker δ_{mn}

33. Evaluate

$\oint_C \frac{e^{iz} \, dz}{z^3}$ for a contour around 0.

34. Evaluate

$$\oint_C \frac{\sin^2 z - z^2}{(z-a)^3}\, dz,$$ where the contour encircles a

35. Evaluate

$$\oint_C \frac{dz}{z\,(2\,z+1)}$$ when C is a unit circle

36. Evaluate

$$\oint_C \frac{dz}{z\,(2\,z+1)^2}$$ when C is a unit circle

37. Calculate Taylor expantion of $\ln(1+z)$

38. Determine the Residue of Function

$$f(z) = \frac{z}{\sin^2(z)} \text{ at } z = \pi$$

and

$$f(z) = \frac{\cot(\pi z)}{z\,(z+1)} \text{ at } z = 0$$

39. Calculate following *integrals* :

$$I = \int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta} \quad |a| < 1$$

$$I = \int_0^{2\pi} \frac{\cos 2\theta}{5 - 4\cos\theta}\, d\theta \quad |a| < 1$$

40. Calculate following *integrals* :

$$I = \int_0^{\infty} \frac{dx}{1 + x^2}$$

$$I = \int_0^{\infty} \frac{\cos x}{1 + x^2}\, dx$$

$$I = \int_0^{\infty} \frac{\sin x}{x}\, dx$$