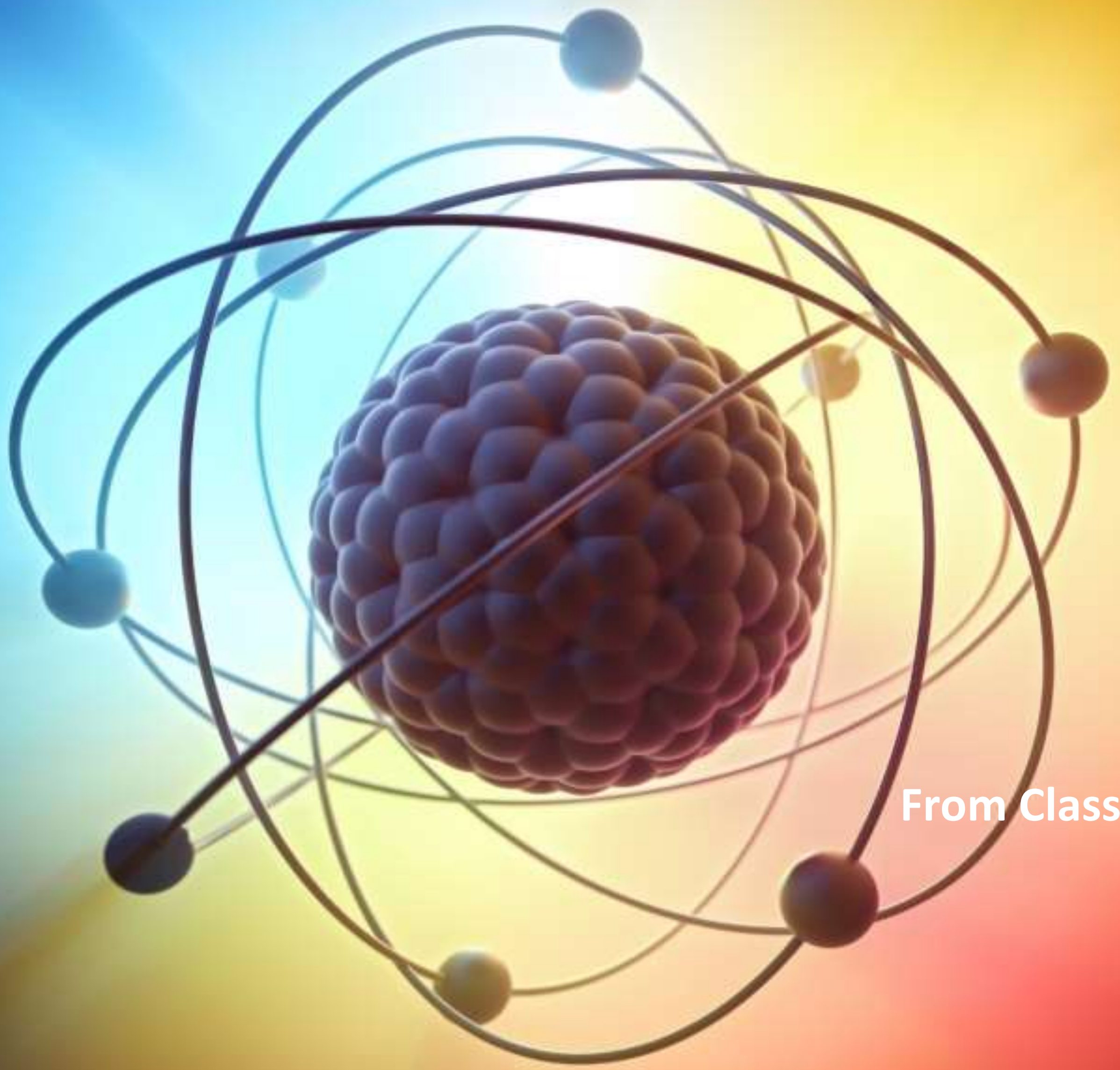

From Classical Mechanics to Modern Foundati

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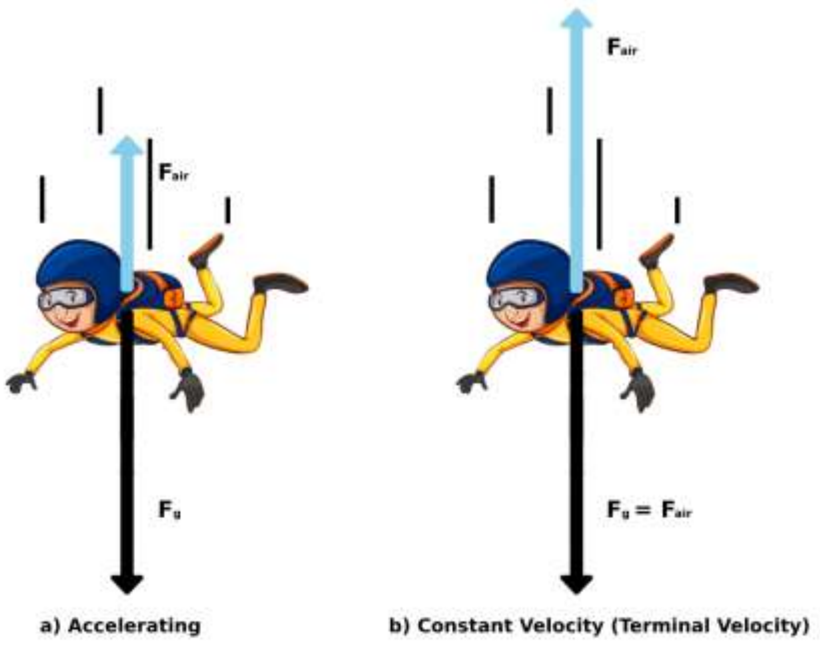
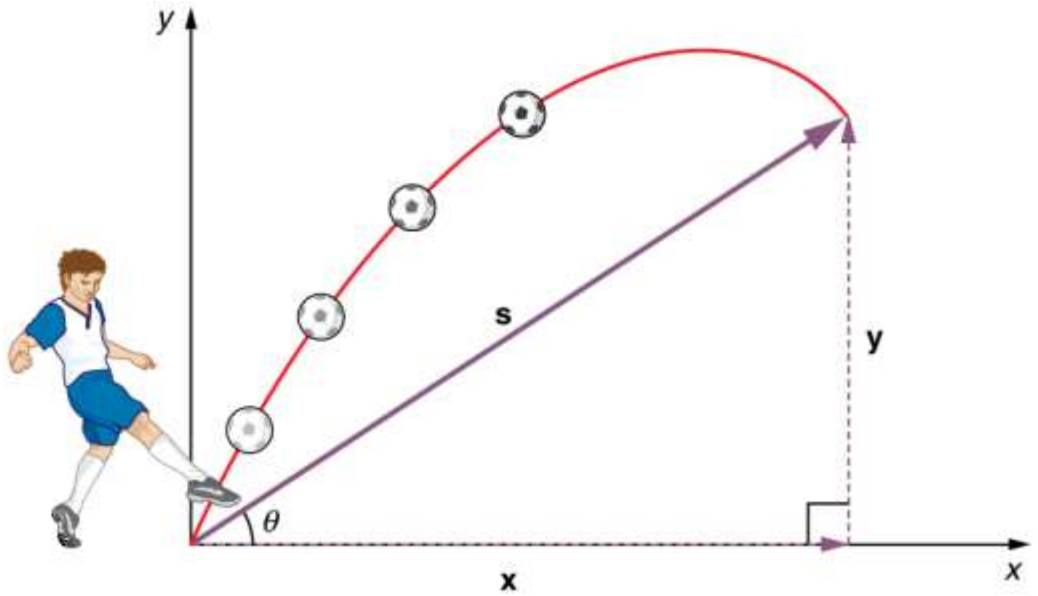
Why Math is Essential in Physics

- Predict behavior of objects
- Solve complex systems (e.g., planetary motion)
- Model quantum systems

Example:

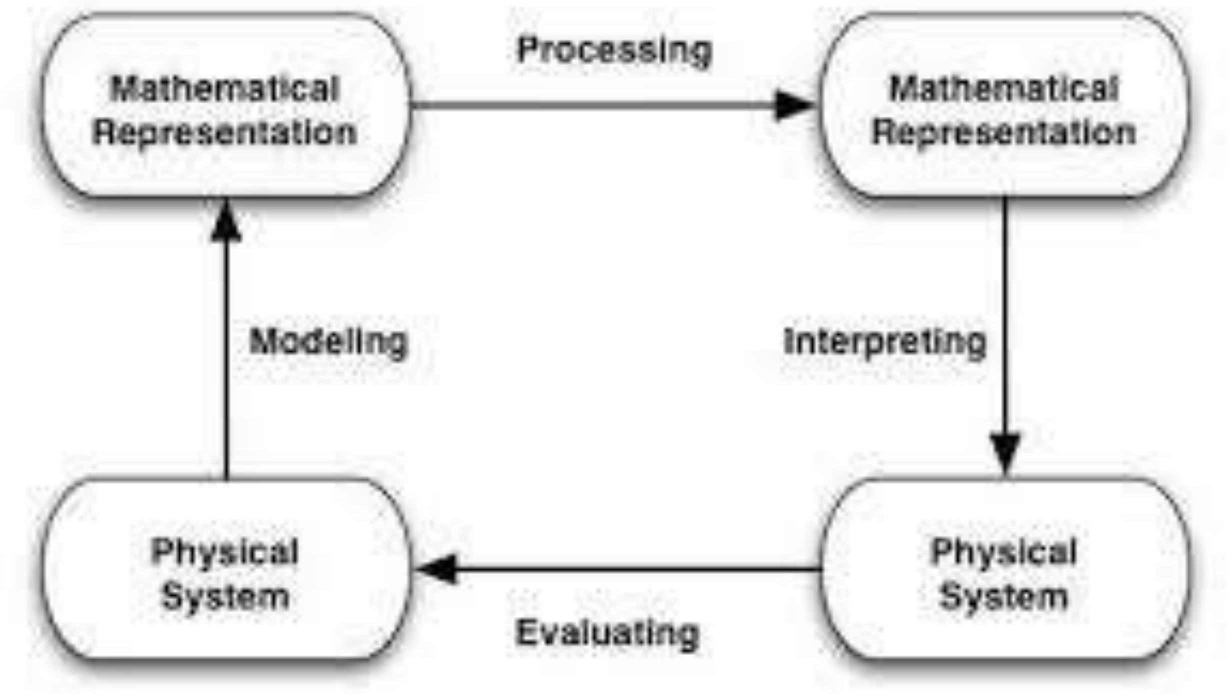
Using differential equations to calculate the trajectory of a projectile





Why Hilbert's Program Matters for Physics

- Physics relies on mathematical structures (calculus, linear algebra, differential equations).
- If mathematics were inconsistent, physical predictions would be unreliable.
- Hilbert's goal was to guarantee the reliability of all mathematical tools used in physics.
- Many physical theories assume the consistency of real numbers, complex numbers, and differential equations.



Example:
Solving classical mechanics problems with confidence in the math.

Connection to Modern Physics

- Quantum theory uses infinite-dimensional Hilbert spaces (named after Hilbert).
- Renormalization problems in quantum field theory reflect the need for strong mathematical foundations.
- Mathematical proofs of stability in quantum systems depend on consistency assumptions Hilbert aimed to secure.

Gödel’s Incompleteness Theorems

Correct Interpretation for Physics

- Gödel’s theorem is about logic, not quantum uncertainty.
- It means: no mathematical system can fully describe all truths about the universe.
- Some physical questions may be fundamentally unprovable using mathematics.

Examples in Physics Related to Undecidability

- Determining long-term behavior of turbulent fluids is related to undecidable problems.
- Some results in general relativity (e.g., singularity detection) map to undecidable logic.
- In quantum gravity, exact consistency cannot be proved from within the theory.

Can Physics Be Fully Axiomatized?

- Hilbert asked whether all of mathematics can be made complete and consistent.
- Gödel showed this is impossible.
- Since physics depends on mathematics → physics cannot be perfectly axiomatized either.
- Some physical truths may forever remain “unprovable” within any theory.

Example:

- Predicting exact turbulent flow structure may be undecidable.

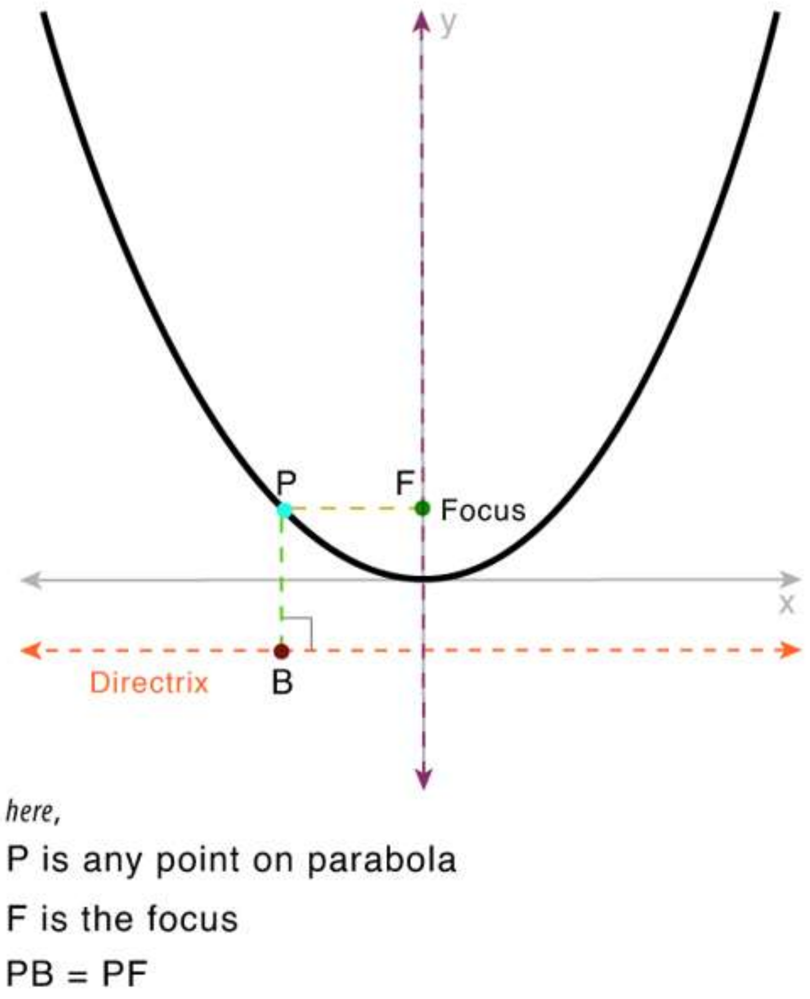
Descartes: Analytical Geometry

- Linked algebra + geometry

Example:
Cartesian coordinates make trajectory calculations easy

Impact on Physics

- Unified algebra + geometry → allowed physical motion to be represented by equations.
- Enabled precise mathematical modeling of trajectories, orbits, and forces.



Newton (Laws of Motion & Calculus)

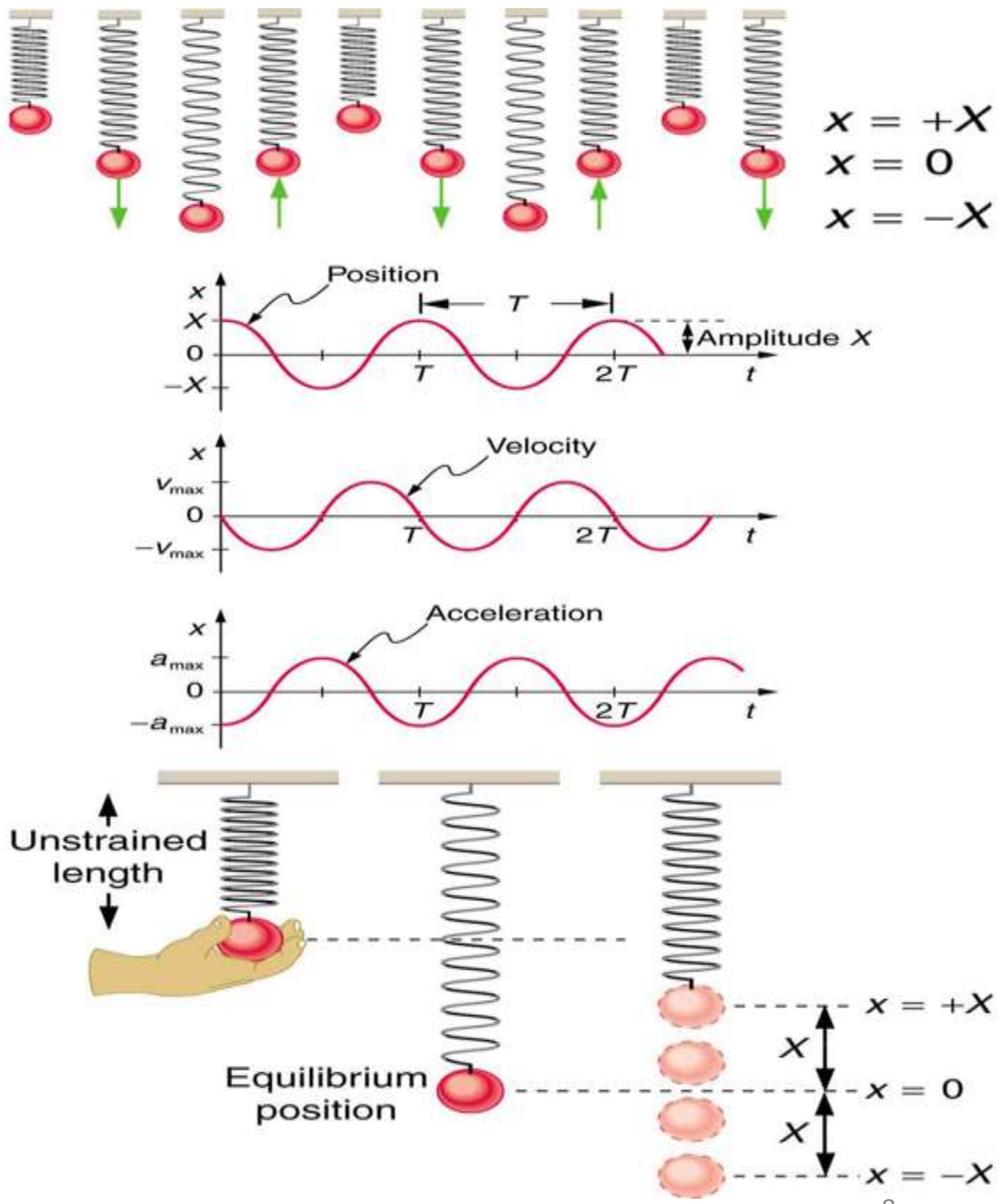
- Developed calculus & laws of motion

Example

- Projectile motion: $y = v_0 t - \frac{1}{2} g t^2$
- Harmonic oscillator: $\frac{d^2x}{dt^2} + \omega^2 x = 0$

Why Newton Changed Physics

- Introduced differential equations as the language of physics.
- Made physics predictive: given initial conditions, future motion can be calculated.
- Connected force → geometry → time evolution.



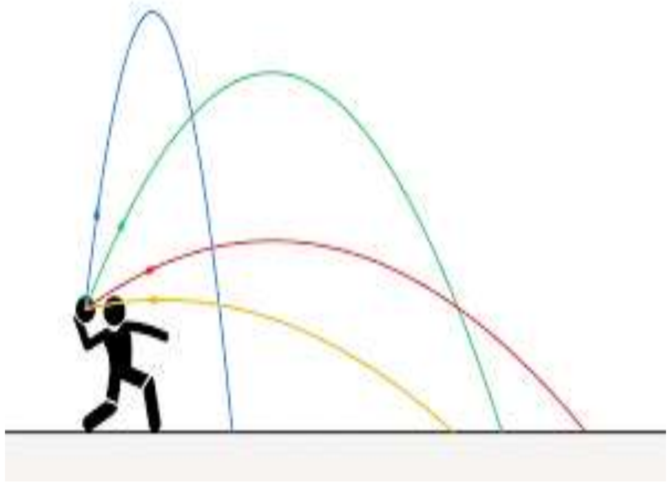
Combined Example: Newton + Calculus

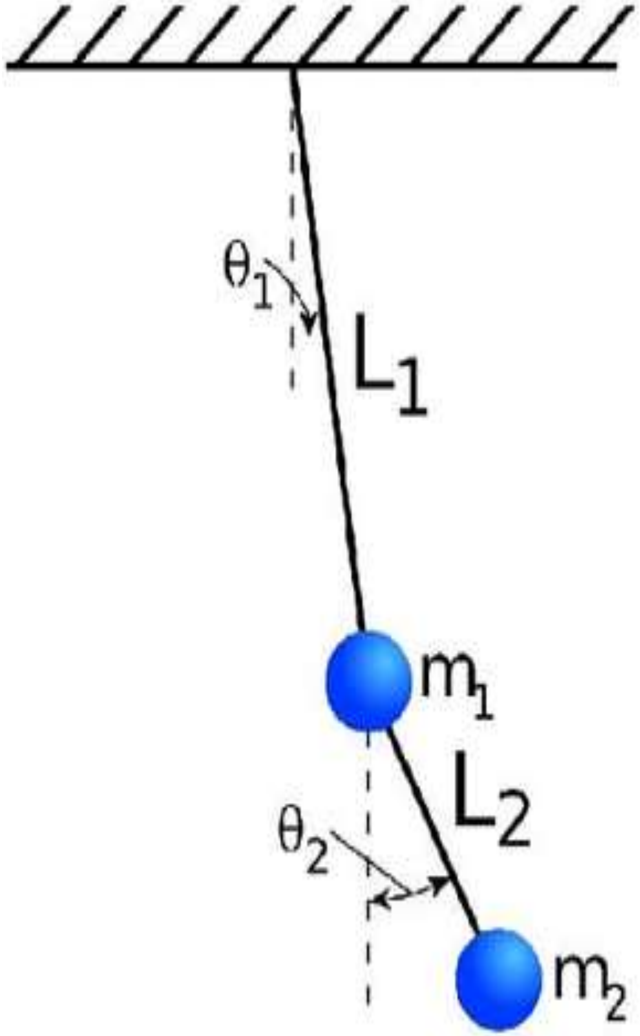
- Predict car acceleration or orbit of a planet

Step 1:
Write equation $F = ma$

Step 2:
Solve differential equation

Step 3:
Predict future motion





Leibniz: Calculus Notation & Mathematical Tools

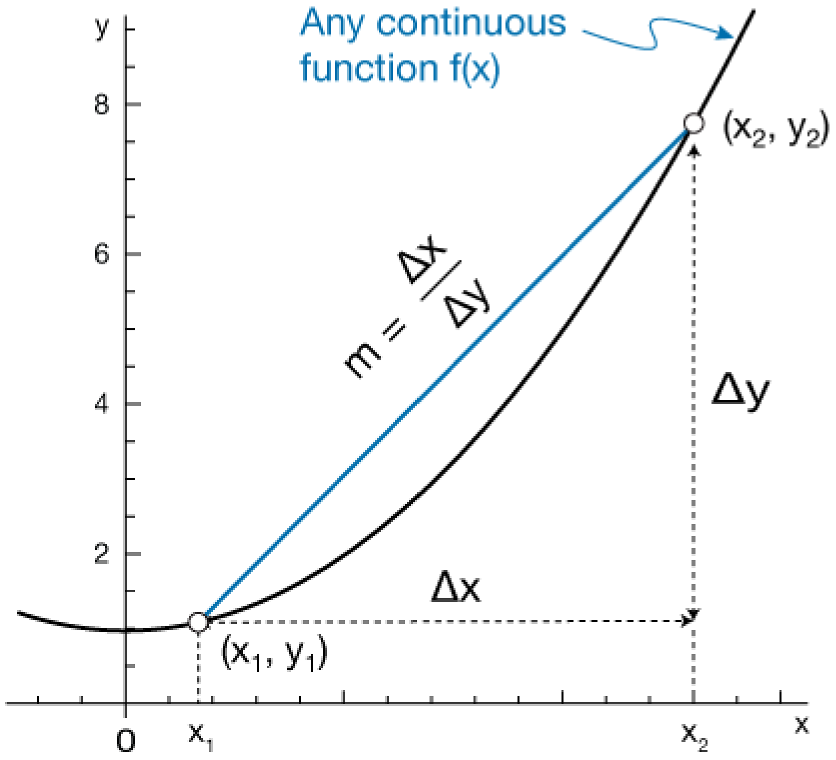
- Developed modern calculus notation
- Easier to solve physics problems systematically

Example

Velocity $v = \frac{dx}{dt}$,acceleration $a = \frac{d^2x}{dt^2}$

Influence on Modern Physics

- His notation (dx/dt, d^2x/dt^2) is still used in mechanics, electromagnetism, and quantum theory.
- Inspired the development of the Lagrangian and Hamiltonian frameworks.
- Early ideas of conservation principles shaped energy and momentum laws.



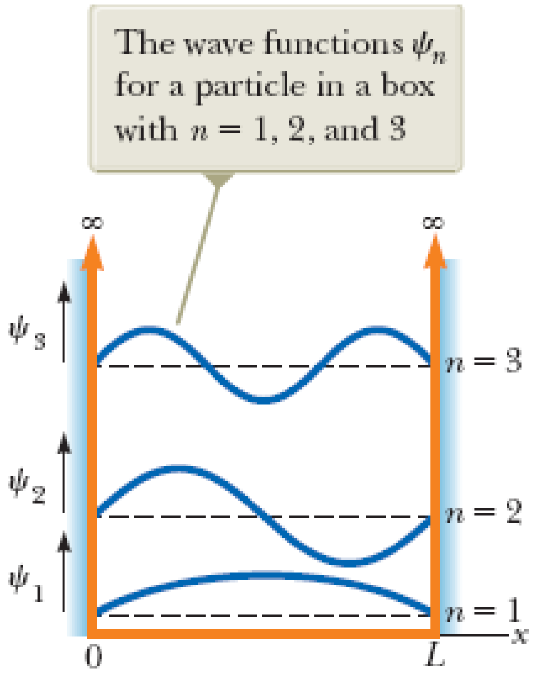
Mathematical Methods in Modern Physics

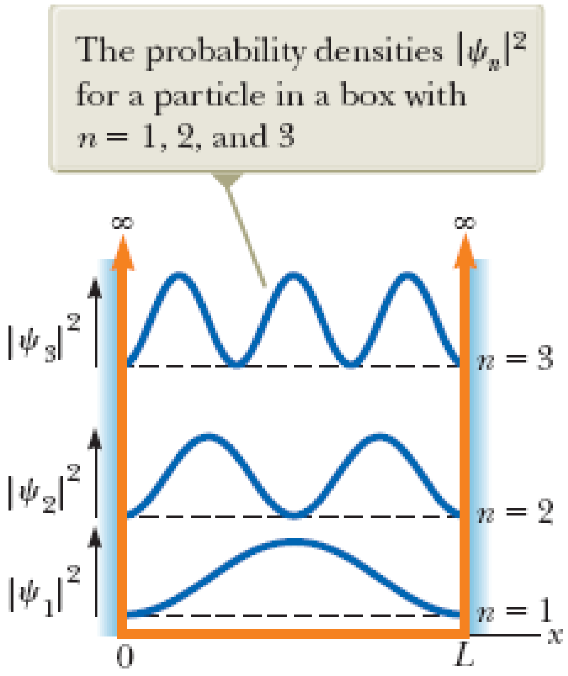
Tools used today:

- Linear algebra → quantum states
- Differential equations → waves, heat, motion
- Complex analysis → electromagnetism, quantum fields

Example: Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$





Application Example (Quantum Mechanics)

- Time-independent Schrödinger equation → predicts energy levels
- Particle in a box / harmonic oscillator

Real-World Application

- GPS accuracy depends on time dilation → solved with math
- Shows direct impact of math in technology

Timeline of Mathematical Methods

- Descartes → Newton → Leibniz → Hilbert → Gödel → Modern physics
- Shows evolution from geometry & calculus → rigorous & modern methods

References

- Hilbert, D., *Foundations of Mathematics*, 1934
- Gödel, K., *On Formally Undecidable Propositions*, 1931
- Kline, M., *Mathematical Thought from Ancient to Modern Times*, 1972
- Griffiths, D. J., *Introduction to Quantum Mechanics*, 2018
- Misner, C. W., Thorne, K. S., Wheeler, J. A., *Gravitation*, 1973