

Homework 10 (20 points each)

1. Calculate $\oint_C z^n dz$ for integers $n \geq -1$

2. Prove Cauchy's Integral Theorem

3. Calculate

$\oint_C \frac{dz}{z^2 + z}$ for circle C defined by $|z| = R > 1$

4. Show that

$\oint_C z^{m-n-1} dz$ where m and n are integers is

Kronecker δ_{mn}

5. Evaluate

$\oint_C \frac{e^{iz} dz}{z^3}$ for a contour around 0 .

6. Evaluate

$\oint_C \frac{\sin^2 z - z^2}{(z - a)^3} dz$, where the contour encircles a

7. Evaluate

$\oint_C \frac{dz}{z(2z + 1)}$ when C is a unit circle

8. Evaluate

$$\oint_C \frac{dz}{z(2z+1)^2} \text{ when } C \text{ is a unit circle}$$

9. Derive Cauchy Inequality and prove the fundamental theorem of algebra.