Homework 7 (20 points each)

1. Show that

$$\int \vec{A}(r) \cdot \vec{\nabla} f(r)\, d^3 r = -\int f(r) \left(\vec{\nabla} \cdot \vec{A}(r)\right) d^3 r$$

$$\int \vec{V}(r) \cdot \left(\vec{\nabla} \times \vec{A}(r)\right) d^3 r = \int \vec{A}(r) \cdot \left(\vec{\nabla} \times \vec{V}(r)\right) d^3 r$$

2. Scetch the Gauss Theorem

3. Show that

$$\oint_{\delta V} F(r)\, d\vec{\sigma} = \int_{V} \vec{\nabla} F(r)\, d^3 r$$

$$\oint_{\delta V} d\vec{\sigma} \times \vec{V}(r) = \int_{V} \vec{\nabla} \times \vec{V}(r)\, d^3 r$$

4. Scetch the Stokes' theorem

5. Show that

$$\int_{S} d\vec{\sigma} \times \vec{\nabla} \phi(r) = \oint_{\delta S} \phi(r)\, d\vec{r}$$

$$\int_{S} \left(d\vec{\sigma} \times \vec{\nabla}\right) \times \vec{V}(r) = \oint_{\delta S} d\vec{r} \times \vec{V}(r)$$

6. Express Maxwell equations through scalar and vector potentials using Lorentz gauge

7. Derive Gauss' Law

8. Show that $\nabla^2 \left(\frac{1}{r}\right) = -4\pi\delta(r)$