

Homework 4

1. (40 points) Prove the following identities

$$(a) (t^a)_m^l (t^a)_k^i = \left(\frac{1}{2}\right) \delta_m^i \delta_k^l - \left(\frac{1}{2N}\right) \delta_k^i \delta_m^l$$

$$(b) t^a t^a = \left(\frac{N^2 - 1}{2N}\right) \mathbf{I}$$

$$(c) \text{Tr} (t^a t^b t^a t^c) = - \left(\frac{1}{4N}\right) \delta^{bc}$$

$$(d) f^{abc} f^{abd} = N \delta^{cd}$$

2. (40 Optional) Prove the following identities

$$(a) t^a t^b t^a = - \left(\frac{1}{2N}\right) t^b$$

$$(b) i f^{abc} t^b t^c = - \left(\frac{N}{2}\right) t^a$$

$$(c) f^{adg} f^{bed} f^{cge} = - \left(\frac{N}{2}\right) f^{abc}$$

$$(d) f^{abc} f^{ade} f^{bdf} f^{ceg} = \left(\frac{N^2}{2}\right) \delta^{fg}$$

3. (40 points) Derive Yang - Mills Lagrange Density using local gauge invariance with respect to SU (N) symmetry