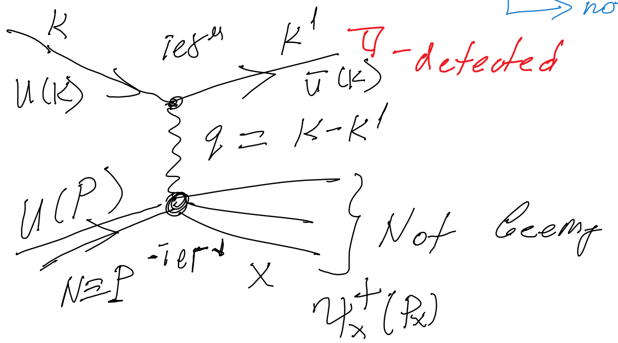
Lecture10

Wednesday, October 25, 2017 3:50 PM

Deep Inelastic Inclusive Processes.

① Inclusive Process

$e + N \longrightarrow e' + X$ ⟶ not measured



Kinematics

$K = (\varepsilon, \vec{K})$

$K' = (\varepsilon', \vec{K}')$

$$K + P_{in} = K' + P_X$$

$$\underbrace{K - K'}_{q} + P_{in} = P_X$$

$$q + P_{in} = P_X$$

$$q = (q_0, \vec{q})$$

$$q_0 = (\varepsilon - \varepsilon')$$

$$\vec{q} = \vec{K} - \vec{K}'$$

$$q = \sqrt{K^2 - 2KK'\cos\theta + K'^2}$$

$$q^2 = q_0^2 - |\vec{q}|^2 = -Q^2 =$$

$$= -4\varepsilon\varepsilon' \sin^2\frac{\theta}{2}$$

⇒ Cross Section of Inelastic Scattering

$\bar{u}(k')\gamma^\mu u(k)$

$q_\nu J_e^\nu = 0$

$$M = \frac{e^2}{\ } \; J_e^\mu(k'k) \; J_{H,\nu}(P_X, P) = ?$$

Cont

$\boxed{q_\mu J_{N}^{\mu} = 0}$

$$|\overline{M}|^2 = \frac{e^4}{q^4}\frac{1}{2}L^{\mu\nu}\cdot\frac{1}{2S_A+1}H_{\mu\nu} =$$

$$4\pi M_N \frac{e^4}{q^4}\frac{1}{2}L^{\mu\nu}\frac{1}{2S_A+1}\frac{H^{\mu\nu}}{4\pi M_N}$$

$$W^{\mu\nu} = \frac{1}{4\pi M_N}\frac{1}{2S_A+1}\sum_{P_X}\sum_{spin} J_A^{\mu\,+}(P)\, J_A^{\nu}(SP)\,(2\pi)^4\delta^4(q+P_N-P_X)\times\frac{d^3P_X}{(2\pi)^3 2E_X}$$

$$W^{\mu\nu} = \left[-g^{\mu\nu} - \frac{q^\mu q^\nu}{Q^2}\right]W_1(M_N^2, M_X^2, Q^2, P_N\cdot q) +$$

$$\left[P_A^\mu + \frac{P_A\cdot q}{Q^2}q^\mu\right]\left[P_A^\nu + \frac{P_A\cdot q}{Q^2}q^\nu\right]\frac{W_2}{M^2}(M_N^2, M_X^2, Q^2, P_N\cdot q)$$

$\Rightarrow$ Why only two structure functions?

In general $g^{\mu\nu}A_1 + q^\mu q^\nu A_2 + P_N^\mu P_N^\nu A_3$

$$\left(P_A^\mu q^\nu + P^\nu q^\mu\right)A_4 = W^{\mu\nu}$$

$\Rightarrow$ Restrictions

1. $W^{\mu\nu} = W^{\nu\mu}$ – symmetric
2. $q^\mu W^{\mu\nu} = q^\nu W^{\mu\nu} = 0$
3. Time reversal symmetry $p^\mu \to -p^\mu$, $q^\mu \to -q^\mu$

We already know

$$L_{\mu\nu} = \frac{1}{2}\mathrm{Tr}(\not{k}'+m)\gamma^\mu(\not{k}+m)\gamma^\nu =$$

$$= 4\left[\left(k_\mu - \frac{q_\mu}{2}\right)\left(k_\nu - \frac{q_\nu}{2}\right)\right] + Q^2\left[-g^{\mu\nu} - \frac{q^\mu q^\nu}{Q^2}\right]$$

show that it can be written in this form

$$\Rightarrow L_{\mu\nu} W^{\mu\nu} = \Big(4 (k_\mu - \tfrac{q}{2})(k_\nu - \tfrac{q}{2}) + Q^2 \big[-g^{\mu\nu} - \tfrac{q^\mu q^\nu}{Q^2} \big] \Big)$$

$$\times \Big(W_1 \big[-g^{\mu\nu} - \tfrac{q^\mu q^\nu}{q^2} \big] + (P^\mu + \tfrac{Pq}{Q^2} q^\mu)(P^\nu + \tfrac{Pq}{Q^2} q^\nu) \tfrac{W_2}{M^2} = \Big)$$

$\Rightarrow$ Considering term $\sim W_1$

$$4 W_1 \Big[-(k - \tfrac{q}{2})^2 + \tfrac{Q^2}{4} \big[4 + \tfrac{q^2}{Q^2} \big] \Big] = \qquad \tfrac{3}{4} - \tfrac{1}{4} = \tfrac{1}{2}$$

$$4 W_1 \Big[- \underset{= 0}{k^2} + \overset{= q^2/2}{k \cdot q} - \tfrac{q^2}{4} + \tfrac{3Q^2}{4} \Big] = 4 W_1 \Big[\tfrac{Q^2}{4} + \tfrac{3Q^2}{4} \Big] =$$

$$= 2Q^2 W_1$$

$\Rightarrow$ Considering $\sim W_2$ term

$$\frac{W_2}{M^2} \big[P^\mu - \tfrac{Pq}{q^2} q^\mu \big] \big[P^\nu - \tfrac{Pq}{q^2} q^\nu \big] \times \Big[4 (k_\mu - \tfrac{q_\mu}{2})(k_\nu - \tfrac{q_\nu}{2}) + Q^2 \big(-g_{\mu\nu} + \tfrac{q_\mu q_\nu}{Q^2} \big) \Big)$$

$$= \frac{W_2}{M^2} \Big[4 \big[Pk - \tfrac{Pq}{q^2} \overset{q^2/2}{qk} \big]^2 - Q^2 \big(P^\mu \cancel{+} \tfrac{Pq}{q^2} q^\mu \big)^2 \Big] =$$

$$= \frac{W_2}{M^2} \Big[4 \big(Pk - \tfrac{Pq}{2} \big)^2 - Q^2 \big(M^2 \cancel{+} \tfrac{2 (Pq)^2}{q^2} + \tfrac{(Pq)^2 q^2}{q^4} \big) \Big)$$

$$= \frac{W_2}{M^2} \Big[4 \big(\tfrac{Pk}{2} + \tfrac{Pk'}{2} \big)^2 - Q^2 \big[M^2 + \tfrac{(Pq)^2}{Q^2} \big] \Big] =$$

target at rest

$$= \frac{W_2}{M^2} \Big[(M (E + E'))^2 - Q^2 \big(M^2 + \tfrac{M^2 q_0^2}{Q^2} \big) \Big).$$

$$= \frac{W_2 M^2}{M^2} \big(E^2 + 2EE' + E'^2 - q_0^2 \big)$$

$$= W_2 \left(E^2 + 2EE' + E'^2 - E^2 + 2EE'\cos\theta - E'^2 \right)$$

$$= W_2\, 4EE'\cos^2\frac{\theta}{2}$$

$$\boxed{\begin{aligned} L^{\mu\nu} W_{\mu\nu} &= 2Q^2 W_1 + W_2\, 4EE'\cos^2\frac{\theta}{2} = \\ &= 4EE'\cos^2\frac{\theta}{2}\left[W_2 + 2\tan^2\frac{\theta}{2} W_1 \right] \end{aligned}}$$

⇒ Differential Cross section

$$d\sigma = \frac{4\pi M\, e^4}{Q^4\, 4E\cdot M}\, 4EE'\cos^2\frac{\theta}{2}\left[W_2 + 2\tan^2\frac{\theta}{2}W_1\right]\frac{E'\,dE'\,d\Omega}{(2\pi)^3\, 2}$$

$$8\pi^3$$

$$= \frac{e^4}{\left(4EE'\sin^2\frac{\theta}{2}\right)^2 E}\;\frac{E'\, 4EE'\cos^2\frac{\theta}{2}}{8\pi^2 \times 2}\left[W_2 + 2\tan^2\frac{\theta}{2}W_1\right] dE'\,d\Omega$$

$$= \frac{\alpha^2\cos^2\frac{\theta}{2}}{4E^2\sin^4\frac{\theta}{2}}\left[W_2 + 2\tan^2\frac{\theta}{2}W_1\right] dE'\,d\Omega'$$

$$\boxed{\frac{d\sigma}{dE'\,d\Omega} = \sigma_{MOTT}\left[W_2 + 2\tan^2\frac{\theta}{2}W_1\right]}$$

W_1 W_2 invarian function of scalars

M^2 M_X^2 pq Q^2

or x Q^2

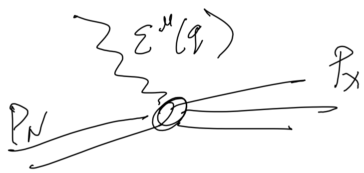
Inelastic Scattering of Virtual Photon

⇒ It is sometimes useful to treat the virtual photon as a particle and consider the reaction

$$\boxed{\gamma^* + N \to X}$$

When γ^* is a virtual photon having energy q_0

— Feynman diagram for this process



$$-iM = \psi^\dagger(P_X)\left[-ie\Gamma^\mu\right]U_{S_N}(P_N)\,\varepsilon^\mu$$

$$M = e\, J_N^\mu(P_X,P_N)\,\varepsilon^\mu$$

$$\overline{|M|^2} = e^2 \sum_X \frac{1}{2}\sum_{S_X S_N} J_N^{\mu\,\dagger}(P_X,P_N)\, J_N^\nu(P_X,P_N)\,\varepsilon^{\mu\,\dagger}\varepsilon^\nu$$
$$(2\pi)^4\delta^4(q+P_N-P_X)\frac{d^3P_X}{2E_X(2\pi)^3}$$

$$= \sum_X \overline{H}^{\mu\nu}(2\pi)^4\delta^4(q+P_N-P_X)\frac{d^3P_X}{2E_X(2\pi)^3}\,\varepsilon^{\mu\dagger}\varepsilon^\nu$$

$\Rightarrow$ But we define

$$W^{\mu\nu} = \frac{1}{4\pi M_N}\sum_X H^{\mu\nu}(P_X,P_N)(2\pi)^4\delta^4(q+P_N-P_X)\frac{d^3P_X}{2E_X(2\pi)^3}$$

Thus

$$\overline{|M|^2} = 4\pi M_N e^2 W^{\mu\nu}\varepsilon^\mu\varepsilon^\nu$$

$$\underset{\sigma_{TOT}}{\overset{|||}{d\sigma}} = \frac{1}{4\ \text{Flux}}\,4\pi M_N e^2 W^{\mu\nu}\varepsilon^{\mu\dagger}\varepsilon^\nu$$

if it were real photon with $\vec{K}_{\text{momentum}}$

$$\text{Flux}\ \sqrt{(P_N\cdot K)^2 - \underset{0}{\overset{||}{K^2}} M_N^2} = M_N \cdot \vec{K}$$

also for real photon produced

final mass $\quad \ldots\ \leq 0$

$$W^2 = (P+K)^2 = M_N^2 + 2PK + K^2 = M_N^2 + 2M_N K$$

So we define $\boxed{K = \frac{W^2 - M_N^2}{2M_N}}$

$\Rightarrow$ For virtual photon we use above relation but

$$W^2 = (q+P_N)^2 = M_N^2 + 2q_0 M_N - Q^2$$

$$\boxed{K_V = \frac{2q_0 M_N - Q^2}{2M_N} = q_0 - \frac{Q^2}{2M_N}}$$

$\Rightarrow$ Going back to the cross section

$$\sigma_{TOT} = \frac{\pi e^2}{K} W^{\mu\nu} \varepsilon^\mu \varepsilon^\nu = \frac{4\pi^2}{K} \alpha W^{\mu\nu} \varepsilon^\mu \varepsilon^\nu$$

$\Rightarrow$ To complete the calculation we use

$$W^{\mu\nu} = \left(p^\mu - \frac{pq}{q^2} q^\mu\right)\left(p^\nu - \frac{pq}{q^2} q^\nu\right)\frac{W_2}{M_N^2} + \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1$$

and for the wave function of virtual photon with helicity λ

ε_λ^μ for $\lambda = \pm 1$ $\quad \varepsilon_\pm = \mp\sqrt{\frac{1}{2}}\,(0; 1, \pm i, 0)$

$\lambda = 0$ $\quad \varepsilon_0 = \frac{1}{\sqrt{-q^2}}\left(\sqrt{q_0^2 + Q^2}, 0, 0, q_0\right)$

$\Rightarrow$ Conditions

$$(\varepsilon_\lambda^\mu)^2 = -1 \qquad \varepsilon_\lambda^\mu q^\mu = 0$$

$\Rightarrow$ Now we can write

$$\sigma_{TOT} = \frac{4\pi^2 \alpha}{K} \sum_\lambda \varepsilon_\lambda^{\mu *} \varepsilon_\lambda^\nu W_{\mu\nu}$$

We introduce

$$\sigma_T = \frac{1}{2}\frac{4\pi^2\alpha}{K}\sum_{\lambda=-1,1} \varepsilon_\lambda^{\mu+}\varepsilon_\lambda^{\nu} W_{\mu\nu}$$

$$\sigma_L = \frac{4\pi^2\alpha}{K}\sum_{\lambda=0} \varepsilon_\lambda^{\mu+}\varepsilon_\lambda^{\nu} W_{\mu\nu}$$

$$\sigma_T = \frac{4\pi^2\alpha}{K}\left[\varepsilon_1^{\mu+}\varepsilon_1^{\nu} W^{\mu\nu} + \varepsilon_{-1}^{\mu+}\varepsilon_{-1}^{\nu} W^{\mu\nu}\right]$$

$(*)$

$(*)$ $\varepsilon_1^{\mu+}\varepsilon_1^{\nu} W^{\mu\nu} + \varepsilon_{-1}^{\mu+}\varepsilon_{-1}^{\nu} W^{\mu\nu} =$

I, II

$$= \varepsilon_1^{+x}\varepsilon_1^{x} W^{xx} + \varepsilon_1^{+x}\varepsilon_1^{y} W^{xy} + \varepsilon_1^{+y}\varepsilon_1^{x} W^{yx} + \varepsilon_1^{+y}\varepsilon_1^{y} W^{yy} +$$

$$+ \varepsilon_{-1}^{+x}\varepsilon_{-1}^{x} W^{xx} + \varepsilon_{-1}^{+x}\varepsilon_{-1}^{y} W^{xy} + \varepsilon_{-1}^{+y}\varepsilon_{-1}^{x} W^{yx} + \varepsilon_{-1}^{+y}\varepsilon_{-1}^{y} W^{yy} =$$

$$= \left(\varepsilon_1^{+x}\varepsilon_1^{x} + \varepsilon_{-1}^{x}\varepsilon_{-1}^{x}\right) W^{xx} + \left(\varepsilon_1^{+y}\varepsilon_1^{y} + \varepsilon_{-1}^{y}\varepsilon_{-1}^{y}\right) W^{yy}$$

$$+ \left(\varepsilon_1^{+x}\varepsilon_1^{y} + \varepsilon_1^{+y}\varepsilon_1^{x} + \varepsilon_{-1}^{+x}\varepsilon_{-1}^{y} + \varepsilon_{-1}^{+y}\varepsilon_{-1}^{x}\right) W^{xy}$$

$$\Rightarrow \varepsilon_1^{+x}\varepsilon_1^{x} + \varepsilon_{-1}^{x}\varepsilon_{-1}^{x} = \frac{1}{2} + \frac{1}{2} = 1$$

$$\Rightarrow \varepsilon_1^{+y}\varepsilon_1^{y} + \varepsilon_{-1}^{+y}\varepsilon_{-1}^{y} = \frac{1}{2} + \frac{1}{2} = 1$$

$$\Rightarrow \left(\varepsilon_1^{+x}\varepsilon_1^{y} + \varepsilon_1^{+y}\varepsilon_1^{x} + \varepsilon_{-1}^{+x}\varepsilon_{-1}^{y} + \varepsilon_{-1}^{+y}\varepsilon_{-1}^{x}\right) =$$

$$= +i - i - i + i = 0$$

$$= W^{xx} + W^{yy} = 2W_1(x, Q^2)$$

$$\boxed{\sigma_T = \frac{4\pi^2\alpha}{K} W_1(x, Q^2)}$$

$$\boxed{\quad K \quad}$$

$$\sigma_L = \frac{4\pi^2\alpha}{K}\,\epsilon_0^{\mu\,\dagger}\,\epsilon_0^{\nu}\,W_{\mu\nu}$$

F

$$F = \epsilon_0^{0\,\dagger}\,\epsilon_0^{0}\,W^{00} - \epsilon_0^{0\,\dagger}\,\epsilon_0^{3}\,W^{03} - \epsilon_0^{3\,\dagger}\,\epsilon_0^{0}\,W^{30} + \epsilon_0^{3\,\dagger}\,\epsilon_0^{3}\,W^{33} =$$

$$= \left(\frac{\sqrt{q_0^2+Q^2}}{\sqrt{Q^2}}\right)^2 W^{00} - \left(\frac{2}{Q^2}\left(\sqrt{q_0^2+Q^2}\cdot q_0\right)\right) W^{03} + \frac{q_0^2}{Q^2}W^{33} =$$

$$= \frac{q_0^2+Q^2}{Q^2}\left\{\left[p^0-\frac{pq}{q^2}q^0\right]^2\frac{W_2}{M^2} + W_1\left[-1+\frac{q^{0\,2}}{q^2}\right]\right\}$$

$$-\frac{2}{Q^2}\left(\sqrt{q_0^2+Q^2}\cdot q_0\right)\left\{\left[\left(p^0-\frac{pq}{q^2}q^0\right)\left[-\frac{pq}{q^2}q_v\right]\frac{W_2}{M^2} + W_1\frac{q_0 q_3}{q^2}\right]\right\} =$$

$$+\frac{q_0^2}{Q^2}\left[\frac{M^2 q_0^2 q_v^2}{Q^2}\frac{W_2}{M^2} + W_1\left(1+\frac{q_v^2}{Q^2}\right)\right]$$

$$= \frac{W_2}{M^2}\left[\frac{q_v^2}{Q^2}\left(M+\frac{M q^{0\,2}}{Q^2}\right)^2 - \frac{2q_v q_0}{Q^2}\left(M+\frac{M q_0^2}{Q^2}\right)\frac{M q_0^3 q_v}{Q^2} + \frac{q_0^2}{Q^2}\frac{M^2 q_0^2 q_v^2}{Q^4}\right] \quad \mathrm{I}$$

$$+ W_1\underbrace{\left[-\frac{q_v^2}{Q^2}\frac{q_v^2}{Q^2} + \frac{2q_v q_0}{Q^2}\frac{q_0 q_v}{Q^2} - \frac{q_0^2}{Q^2}\frac{q_0^2}{Q^2}\right]}_{\mathrm{II}} =$$

II

$$\boxed{W_1\frac{1}{Q^4}\left[-q_v^4+2q_0^2q_v^2-q_0^4\right] = -W_1\frac{\left[q_v^2-q_0^2\right]^2}{} = -\frac{W_1}{}}$$

$$Q^4$$

$$\text{I} = \frac{W_2}{M^2} M^2 \left[\frac{\nu^2}{Q^2} \cdot \frac{q_V^4}{Q^4} - 2\frac{q_0^2 q_V^2}{Q^4}\frac{q_V^2}{Q^2} + \frac{q_0^4}{Q^4} q_V^2 \right.$$

$$W_2 \frac{q_V^2}{Q^2} \left[\frac{q_V^4}{Q^4} - \frac{2 q_0^2 q_V^2}{Q^4} + \frac{q_0^4}{Q^4} \right] =$$

$$W_2 \frac{q_V^2}{Q^2}$$

Therefore

$$\boxed{\sigma_L = \frac{4\pi^2\alpha}{K} \left[\frac{q_V^2}{Q^2} W_2(x,Q^2) - W_1(x,Q^2) \right]}$$

In Summary

$$\sigma_T = \frac{4\pi^2\alpha}{K} W_1(\nu, q^2)$$

$$\sigma_L = \frac{4\pi^2\alpha}{K} \left[\frac{q_V^2}{Q^2} W_2(x,Q^2) - W_1(x,Q^2) \right]$$

Using also

$$\frac{d\sigma}{dE' d\Omega} = \sigma_{MOTT} \left[W_2 + 2\tan^2\frac{\theta}{2} W_1 \right]$$

One obtains that

$$\frac{d\sigma}{dE' d\Omega} = \Gamma (\sigma_T + \varepsilon \sigma_L)$$

$$\Gamma = \frac{\alpha K}{2\pi^2 Q^2}\,\frac{E'}{E}\,\frac{1}{1-\varepsilon}$$

$$\varepsilon = \left(1 + 2\frac{q_v^2}{Q^2}\tan^2\frac{\theta}{2}\right)^{-1}$$

⇒ prove

$$W_1 = \frac{1}{\beta}\sigma_T \qquad \beta = \frac{4\pi^2\alpha}{K}$$

$$\frac{1}{\beta}\sigma_L = \frac{q_v^2}{Q^2}W_2 - \frac{1}{\beta}\sigma_T$$

$$W_2 = \frac{Q^2}{q_v^2}\left(\frac{\sigma_T+\sigma_L}{\beta}\right)$$

$$\frac{d\sigma}{dE\,d\Omega} = \sigma_{Mott}\left(\frac{Q^2}{q_v^2}\left(\frac{\sigma_T+\sigma_L}{\beta}\right) + 2\tan^2\frac{\theta}{2}\frac{1}{\beta}\sigma_T\right)$$

$$= \sigma_{Mott}\left(\frac{Q^2}{q_v^2\beta}\frac{\sigma_L}{\beta} + \frac{\sigma_T}{\beta}\left(\frac{Q^2}{q_v^2} + 2\tan^2\frac{\theta}{2}\right)\right)$$

$$= \frac{\sigma_{Mott}}{\beta}\cdot\left(\frac{Q^2}{q_v^2} + 2\tan^2\frac{\theta}{2}\right)\left(\sigma_T + \frac{Q^2/q_v^2}{\frac{Q^2}{q_v^2} + 2\tan^2\frac{\theta}{2}}\sigma_L\right)$$

(*)

$$\boxed{\varepsilon = \frac{Q^2/q_v^2}{\frac{Q^2}{q_v^2} + 2\tan^2\frac{\theta}{2}} = \frac{1}{1 + 2\frac{q_v^2}{Q^2}\tan^2\frac{\theta}{2}}}$$

(*) $$\frac{\sigma_{Mott}}{4\pi^2\alpha}\cdot K\cdot\frac{Q^2}{q_v^2}\left(1 + 2\frac{q_v^2}{Q^2}\tan^2\theta\right) =$$

$$= \frac{\alpha^2 \cos^2\theta/2}{4\pi^2\alpha \cdot 4E^2 \sin^4\frac{\theta}{2}} \cdot K \frac{4EE' \sin^2\frac{\theta}{2}}{q_v^2}\left(1 + \frac{2q_v^2}{Q^2}\tan^2\frac{\theta}{2}\right)$$

$$= \frac{\alpha}{4\pi^2}\frac{E'}{E}\frac{K}{\tan^2\frac{\theta}{2}\, q_v^2}\left(1 + \frac{2q_v^2}{Q^2}\tan^2\frac{\theta}{2}\right)$$

$$= \frac{\alpha K}{2\pi^2 Q^2}\frac{E'}{E}\left(\frac{1 + 2\frac{q_v^2}{Q^2}\tan^2\frac{\theta}{2}}{\frac{2q_v^2}{Q^2}\tan^2\frac{\theta}{2}}\right)$$

$\oplus$

$$\oplus = \frac{1}{\varepsilon}\frac{1}{\left(\frac{1}{\varepsilon} - 1\right)} = \frac{1}{1-\varepsilon}$$

So

$$\oplus = \frac{\alpha K}{2\pi^2 Q^2}\frac{E'}{E}\frac{1}{1-\varepsilon}$$

Therefore

$$\frac{d\sigma}{dE'd\Omega} = \frac{\alpha K}{2\pi^2 Q^2}\frac{E'}{E}\frac{1}{1-\varepsilon}\left(\sigma_T + \varepsilon\sigma_L\right)$$

$\Rightarrow$ Real Photon limit $q^2 = -Q^2 \to 0$

We have

$$W^{\mu\nu} = \frac{W_2}{M^2}\left(p^\mu - \frac{pq}{q^2}q^\mu\right)\left(p^\nu - \frac{pq}{q^2}q^\nu\right) + W_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)$$

$q^2 \to 0$

$$\frac{W_2}{M^2}\frac{(pq)^2}{q^4}q^\mu q^\nu +$$

$W^{\mu\nu}$ — has singularity $\quad \frac{\ }{\mu^2} \ \frac{\ }{q^4}$

$$+ W_1 \frac{q^\mu q^\nu}{q^2}$$

To remove this singularity

$$\frac{W_2}{\mu^2} \frac{(Pq)^2}{q^2} \simeq -W_1$$

i.e. in the Lab $W_2 \frac{q_0^2}{q^2} \simeq -W_1$

or $$\boxed{W_2 \longrightarrow -\frac{q^2}{\nu^2} W_1 + O(q^4)}$$

$\Rightarrow$ Looking at σ_L σ_T

$$\sigma_T = \frac{4\pi^2 \alpha}{K} W_1$$

$$\sigma_L = \frac{4\pi^2 \alpha}{K} \left(\left(1 - \frac{\nu^2}{q^2}\right) W_2 - W_1 \right)$$

$$\sigma_L \Big|_{q^2 \to 0} = \frac{4\pi^2 \alpha}{K} \left(-\frac{\nu^2}{q^2} \underbrace{W_2}_{q^2 \to 0:\ -q^2/\nu^2 W_1} - W_1 \right) \Big| \to 0$$

Therefor in $q^2 \to 0$ Limit

Only $\sigma_T \sim W_1$ survives

while σ_L $\longrightarrow 0$