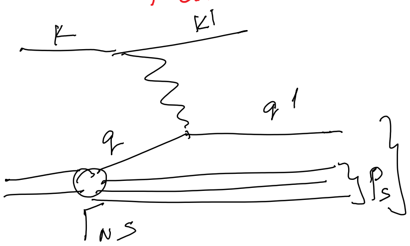
Lecture11

Wednesday, November 1, 2017   1:48 PM

Parton Model



$\times$

$x_B = \dfrac{Q^2}{2q\cdot P_N}$

$(P_N - P_s)^2 - m_q^2$

$\delta^4(q + P_N - P_{N'})\,\dfrac{d^3P_{q'}}{2E_{q'}(2\pi)^3}$

$\dfrac{d^3P_s}{2E_s(2\pi)^3}$

$$-iM = \bar{u}(k')(ie\gamma^\mu)u(k)\frac{[-ig_{\mu\nu}]}{q^2+i\varepsilon}\times$$

$$\times\ \psi^\dagger(P_s)\,\bar{u}(P_{q'})\,[-iee_q\gamma^\nu]\ \otimes$$

$$\otimes\ \frac{\not{P}_q + m_q}{P_q^2 - m_q^2 + i\varepsilon}\,\Gamma^{N\to q,S}\,U_N(P_N)$$

$M_N^2 - 2P_NP_s + M_s^2 - m_q^2$

$(q+P_q)^2 - m_q^2$

$-Q^2 + 2qP_q = 0$

$-Q^2 + x\,2q\cdot P$

Consider

$$\frac{1}{P_q^2 - m_q^2 + i\varepsilon} = \frac{1}{E_q^2 - \vec{P}_q^2 - m^2 + i\varepsilon} =$$

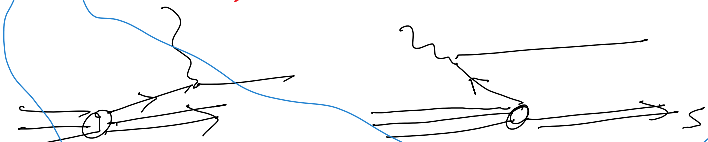
$$E_q^2 - (P_q^2 + m^2 - i\varepsilon)$$

$$= \frac{1}{\left(E_q - \sqrt{P_q^2+m^2 - i\varepsilon}\right)\left(E_q + \sqrt{P_q^2+m^2 - i\varepsilon}\right)} =$$

$$= \frac{1}{\left(E_q - \sqrt{P_q^2+m^2} + i\varepsilon\right)\left(E_q + \sqrt{P_q^2+m^2} - i\varepsilon\right)} =//$$

$$E_q = E_p - E_s = \sqrt{M_p^2 + P_p^2} - \sqrt{M_s^2 + P_s^2}$$

$\longrightarrow$ t-ordered



$$//= \left[\underbrace{\frac{1}{E_q - \sqrt{P_q^2+m^2} + i\varepsilon}}_{a} + \underbrace{\frac{1}{E_q + \sqrt{P_q^2+m^2} - i\varepsilon}}_{b}\right]\frac{1}{2E_q} =$$

$$= \left(\frac{1}{\sqrt{M_p^2+P_p^2} - \sqrt{M_S^2+P_S^2} - \sqrt{m^2+P_q^2}} + \frac{1}{\sqrt{M_p^2+P_p^2} - \sqrt{M_S^2+P_S^2} + \sqrt{m^2+P_q^2}} \right) \frac{1}{2E_q}$$

(a) (b)

$\Rightarrow$ In the Lab frame

$$= \left(\frac{1}{M_p - \sqrt{M_S^2+P_S^2} - \sqrt{m^2+P_q^2}} + \frac{1}{M_p - \sqrt{M_S^2+P_S^2} + \sqrt{m^2+P_q^2}} \right) \frac{1}{2E_q}$$

(a) (b)

If $m \ll M_p, M_S$ Then

and $P_q \ll M_p, M_S$

(a) $\sim$ (b) in the Lab

$\Rightarrow$ Infinite Momentum Frame

$P_p \longrightarrow \infty$

$P_q^z = x P_p \qquad P_S^z = (1-x) P_p$

$P_q^\perp \qquad P_S^\perp = -P_q^\perp$

$$\sqrt{M_N^2+P_p^2} = P_p + \frac{M_N^2}{P_p}$$

$$\sqrt{M_S^2+P_S^2} = \sqrt{(1-x)^2P_p^2 + M_S^2 + P_{q\perp}^2} = (1-x)P_p + \frac{m_S^2+P_{q\perp}^2}{2(1-x)P_p}$$

$$\sqrt{m_q^2+P_q^2} = \sqrt{x^2P_p^2 + m_q^2+P_{q\perp}^2} = xP_p + \frac{m_q^2+P_{q\perp}^2}{2xP_p}$$

(a)
$$\frac{1}{\frac{M_N^2}{P_p} + P_p - (1-x)P - \frac{m_S^2+P_{q\perp}^2}{2(1-x)P} - xP_p - \frac{m_q^2+P_{q\perp}^2}{2xP}}$$

$$= \frac{P_p}{M_N^2 - \frac{m_S^2+P_{q\perp}^2}{2(1-x)} - \frac{m_q^2+P_{q\perp}^2}{2x}}$$

If $P_{q\perp}^2 \ll P_p^2$ x is finite

(a) $\longrightarrow \infty$ $\quad P_p \longrightarrow \infty$ Large:

⑥
$$\frac{1}{\frac{M_N^2}{P_D} + P_D - (1-x)P - \frac{m_s^2 + P_{q\perp}^2}{2(1-x)P} + xP + \frac{m_q^2 + P_{q\perp}^2}{2xP}} \approx$$

$$\approx \frac{1}{2xP_N} \longrightarrow 0 \quad P_N \longrightarrow \infty$$

⇒ Diagram ⓐ Dominates In the infinite momentum frame

using $\not{P}_q + m = \sum_s u(P_q)\bar{u}(P_q)$

$$A_p^\mu = \sum_{s_i} \Psi_{sp}^\dagger(P_s)\, \bar{u}_{s_f}(P_{qf})\, e_q \gamma^\mu u_{s_i}(P_{qi}) \cdot \frac{\bar{u}_{s_i}(P_q)\Gamma\, u(P)}{2E_q(E_p - E_s - E_q^{on})}$$

introduce

$$\Psi_p^{s_i,s_p}(x, P_{q\perp}) = \frac{\Psi_{sp}^\dagger\, \bar{u}_{s_i}(P_q)\Gamma\, u_{s_p}(P)}{2E_q(E_p - E_s - E_q^{on})} \cdot x \cdot 2(2\pi)^3$$

$$A_p^\mu = \sum_{s_i} \bar{u}_{s_f}(P_{qf})\, e_q \gamma^\mu u_{s_i}(P_{qi}) \frac{\Psi^{s_i s_p}(x, P_{qi})}{x\, 2(2\pi)^3}$$

— in the limit of $m_q \to 0$ $s_f = s_i$

⇒ Scattering Amplitude becomes.

$$-iM = \bar{u}(k_f)\, e\gamma^\mu u(k_i) \left(\frac{-ig_{\mu\nu}}{q^2}\right) \times \sum_{s_i} \bar{u}_{s_f}(P_{qf})\, (-ie_q)\gamma^\nu u_{s_i}(P_{qi}) \frac{\Psi^{s_i,s_p}(xP, P_{q\perp})}{x\sqrt{2(2\pi)^3}}$$

$$\overline{|M|}^2 = L^{\mu\nu} H^{\mu\nu} \cdot \frac{e^4}{q^4}$$

$$\bar{H}^{\mu\nu} = \frac{1}{\ldots} \sum \sum \left[\sum \bar{u}_{s_f}(P_{qf})\, e_q \gamma^\mu u_{s_i}(P_{qi}) \frac{\Psi^{s_i s_p}(x, P_q)}{\ldots (2\pi)^3} \right]^\dagger$$

$$\nu \quad S_p \; s_f \, | \, s_i \qquad A(S_f S_i) \qquad \phi^+(s_i, s_p) \qquad \times\, u(m)^3)$$

$$\left(\sum_{s_i'} \bar{U}_{s_f}(P_{q_f}) \, e_q \gamma^\nu U_{s_i'}(P_{q_i}) \, \Psi^{s_i' s_p}(x, P_p) \right)$$

$$A^\nu(S_f, S_i) \qquad \times\, u(m)^3 \quad \phi(s_i', s_p)$$

$$= \frac{1}{2} \sum_{s_p} \sum_{s_f} \left(\sum_{s_i} \sum_{s_i'} \phi^+(s_i, s_p) \, A^{+\mu}(S_f S_i) \, A^\nu(S_f, S_i') \, \phi(S_i', S_p) \right) = //$$

using

$$\sum_{S_p} \phi^+(S_i, S_p) \, \phi(S_i', S_p) = \delta_{s_i s_i'}$$

$$// \; \frac{1}{2} \sum_{S_p} \sum_{S_f} \sum_{S_i} \left(A^{\mu +}(S_f, S_i) \, A^\nu(S_f, S_i) \, \phi^2(S_i S_p) \right)$$

$$\text{using} \sum_{S_p} \phi^2(S_i S_p) = \sum_{S_p} \phi^2(-S_i\, S_p)$$

$$// = \frac{1}{2} \sum_{S_p} \sum_{S_f} \sum_{S_i} \left(A^{\mu +}(S_f, S_i) \, A^\nu(S_f S_i) \times \right.$$

$$\times \sum_{S_i} \phi^2(S_i, S_p) =$$

$$= \frac{1}{2} \sum_{S_f} \sum_{S_i} A^{\mu +}(S_f, S_i) \, A^\nu(S_f, S_i) \times$$

$$\times \; \frac{1}{2} \sum_{S_p} \sum_{S_i} \phi^2(S_i, S_p).$$

One obtains

$$\frac{1}{2}\mathrm{Tr}\left(\not{P}_{qf}\gamma^\mu \not{P}_{qi}\gamma^\nu\right)$$

$$\bar{H}^{\mu\nu} = \sum_q e_q^2 \frac{1}{2}\sum_{s_f}\sum_{s_i}\left(\bar{U}_{sf}(P_{qf})\gamma^\mu U_{si}(P_{qi})\right)\left(U_{sf}(P_{qf})\gamma^\nu U_{si}(P_{qi})\right)$$

$$\frac{1}{2}\sum_{s_f}\sum_{s_i}\frac{\left|\psi^{s_i s_f}(x, P_{q1})\right|^2}{x^2\, 2(2\pi)^3} \qquad \left|\bar{\psi}(x, P_{q1})\right|^2$$

Using above for $|\bar{M}|^2$ one obtains

$$|\bar{M}|^2 = \bar{L}^{\mu\nu}\bar{H}_{\mu\nu}\frac{e^4}{q_4} =$$

$$= \bar{L}^{\mu\nu}\sum_{eq} e_q^2\, \frac{1}{2}\mathrm{Tr}\left(\not{P}_{qf}\gamma^\mu \not{P}_{qi}\gamma^\nu\right)\cdot\frac{\left|\bar{\psi}(x, P_{qi})\right|^2}{x^2\, 2(2\pi)^3}$$

From the previous lecture we have

$$W_{\mu\nu} = \frac{1}{4\pi M_N}\bar{H}^{\mu\nu}(2\pi)^4\delta^4(q + P_N - P_{qf} - P_{qs})\times$$

$$\frac{d^3 P_{qf}}{2E_f(2\pi)^3}\,\frac{d^3 P_{qs}}{2E_s(2\pi)^3}.$$

writing $\frac{d^3 P_{qf}}{2E_f} = \delta(P_{qf}^2 - m_f^2)\, d^4 P_{qf}$

and integrating by $d^4 P_{qf}$ one obtains

$$W_{\mu\nu} = \frac{1}{4\pi M_N}\bar{H}^{\mu\nu}(2\pi)\,\delta(P_{qf}^2 - m_f^2)\frac{d^3 P_{qs}}{2E_s(2\pi)^3} \Rightarrow$$

$4\pi M_N$

Consider $p_{q_f}^2 - m_q^2 = (p_{q_i}+q)^2 - m_q^2$

$= 2p_{q_i}q - Q^2 = ||$

in the IMF

$p_{q_i}q = x\, p_N q$

$|| = x\, 2p_N q - Q^2$

$$|| = \frac{2\pi}{4\pi M_N} \bar{H}^{\mu\nu}\, \delta(x 2p_N q - Q^2)\, \frac{dx_s}{x_s} \frac{d^2 p_{qs\perp}}{2(2\pi)^3}$$

$$= \frac{1}{2M_N} \bar{H}^{\mu\nu} \frac{1}{2p_N q} \delta(x - x_{Bj}) \frac{dx_s}{x_s} \frac{d^2 p_{qs\perp}}{2(2\pi)^3} = ||$$

where

$$\boxed{x_{Bj} = \frac{Q^2}{2p_N q} \overset{\text{Lab}}{=} \frac{Q^2}{2M q_0}}$$

$\Rightarrow$ Inserting $\bar{H}^{\mu\nu}$

$$= || = \frac{1}{2M_N} \sum_q e_q^2 \frac{1}{2} \mathrm{Tr}(\not{p}_{q_f} \gamma^\mu \not{p}_{q_i} \gamma^\nu) \frac{|\overline{\psi(x, p_{qs})}|^2}{x^2\, 2(2\pi)^3}$$

$$\times \frac{1}{2p_N q} \delta(x - x_{Bj}) \frac{dx_s}{x_s} \frac{d^2 p_{qs}}{2(2\pi)^3}$$

Consider

$$\frac{1}{4} \mathrm{Tr}(\not{p}_{q_f} \gamma^\mu \not{p}_{q_i} \gamma^\nu) = 2\left(p_{q_f}^\mu p_{q_i}^\nu + p_{q_f}^\nu p_{q_i}^\mu - (p_{q_f} p_{q_i}) g^{\mu\nu}\right)$$

$=11$

$\boxed{\text{using } P_{q_f}^{\mu} = P_{q_i}^{\mu} + q^{\mu}}$

$$11 = 4\left(P_{q_i}^{\mu} + \frac{q^{\mu}}{2}\right)\left(P_{q_f}^{\nu} + \frac{q^{\nu}}{2}\right) + Q^2\left(-g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2}\right)$$

$$W^{\mu\nu} = \frac{1}{2M_N}\sum_q e_q^2\left[4\left(P_{q_i}^{\mu} + \frac{q^{\mu}}{2}\right)\left(P_{q_f}^{\nu} + \frac{q^{\nu}}{2}\right) + Q^2\left(-g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2}\right)\right]$$
$$\frac{1}{2P_N q X^2}\ \frac{|\Psi(X, P_{q_{1i}})|^2}{2(2\pi)^3}\,\delta(X - X_{Bj})\frac{dX_s}{X_s}\frac{d^2P_{q_{s\perp}}}{2(2\pi)^3}$$

On the other hand, by definition

$$W^{\mu\nu} = \left(P_N^{\mu} + \frac{P_N q}{Q^2}q^{\mu}\right)\left(P_N^{\nu} + \frac{P_N q}{Q^2}q^{\nu}\right)\frac{W_2^N}{M_N^2} + \left[-g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2}\right]W_1^N$$

Comparing two

$$\boxed{W_1^N} = \frac{1}{2M_N}\sum_q e_q^2\,\frac{Q^2}{2P_N q X^2}\,\frac{|\Psi(X, P_{q_i})|^2}{} \times$$
$$\delta(X - X_{Bj})\frac{dX_s}{X_s}\frac{d^2P_{q_s}}{2(2\pi)^3} =$$

$$= \boxed{\frac{1}{2M_N}\sum_q e_q^2 f_q(X)}$$

where

$$f_q(X) = |\Psi(X, P_{q_i})|^2\,\delta(1 - X_{q_i} - X_{q_s})\times$$
$$\times\,\delta(X - X_{Bj})\,\delta^2(P_{q_{i\perp}} + P_{q_{s\perp}})$$

$$\frac{dx_i}{x_i}\frac{dP_{q_{i\perp}}^2}{2(2\pi)^3} \quad \frac{dx_s}{x_s}\frac{dP_{q_{s\perp}}^2}{2(2\pi)^3}$$

$\Rightarrow$ Consider now $g_{\mu\nu}W^{\mu\nu}$

$$q_{\mu\nu}W^{\mu\nu} = \left(P_N^\mu + \frac{P_N q}{Q^2} q^\mu\right)^2 \frac{W_2^N}{M_N^2} - 3W_1^N$$

$$g_{\mu\nu}W^{\mu\nu} = \frac{1}{2M_N}\sum_q e_q^2\left(4\left(P_{q_i}^\mu + \frac{q^\mu}{2}\right)^2 - 3Q^2\right)\frac{1}{2P_N q x} f_q(x)$$

$$\left(P_N^2 + \frac{2P_N q\, P_N q}{Q^2} - \frac{(P_N q)^2}{Q^2}\right)\frac{W_2^N}{M_N^2} = \qquad 2P_N q = Q^2$$

$$= \frac{1}{2M_N}\sum_q e_q^2 \cdot 4\left(m_q^2 + P_q q - \frac{Q^2}{4}\right)\frac{1}{P_N q x} f_q(x)$$

$$= \frac{(P_N q)^2}{Q^2}\frac{W_2^N}{M_N^2} = \frac{1}{2m_N}\sum e_q^2 \cdot \frac{Q^2}{4}\cdot 4\,\frac{1}{2P_N q}\frac{f_q(x)}{x}$$

$$q_0 W_2^N \cdot \frac{P_N q}{Q^2} = \sum e_q^2 \frac{1}{2} f_q(x)$$

$$\boxed{q_0 W_2^N = \sum e_q \frac{Q^2}{2P_N q} f_q(x) = \sum e_q\, x f_q(x)}$$

Introducing $F_1 = m_N W_1$

$$F_2 = q_0 W_2$$

$$\frac{d\sigma}{dE' d\Omega} = \sigma_{Mott}\left[W_2 + 2\tan^2\frac{\theta}{2} W_1 \right] =$$

$$= \sigma_{mott}\left[\frac{F_2(x)}{q_0} + 2\tan^2\frac{\theta}{2} \frac{F_1(x)}{m_N} \right]$$

$$= \frac{\sigma_{Mott}}{q_0}\left[F_2(x) + 2\tan^2\frac{\theta}{2} \frac{F_1(x) q_0}{m_N} \right]$$

If $\tan^2\frac{\theta}{2} \ll 1$

$$\frac{d\sigma}{dE' d\Omega} \Big/ \frac{\sigma_{mott}}{q_0} = F_2(x_{Bj})$$

should depend only on

$$x_{Bj} = \frac{Q^2}{2 m q_0}$$

but not Q^2 and q_0

$\Rightarrow$ From the considered parton model

$$m W_1 = \frac{1}{2} \sum e_q^2 f_q(x_B) = F_1$$

$$q_0 W_2 = \sum_q e_q^2 x_B f_q(x_B) = F_2$$

$$\boxed{2xF_1 = F_2 = \sum_i e_i^2 x f_q(x)}$$

→ Callan Gross Relation

⇒ Other Representations:

Introduce $y = \frac{P_\nu q_\nu}{P_\nu k} \underset{\text{Lab}}{=} \frac{q_0}{E}$

Show

$$dE' d\Omega = \frac{\pi}{EE'} dQ^2 d q_0 = \frac{2ME}{E'} \pi y \, dx \, dy$$

$0 < x < 1 \qquad y \leq 1$

Show:

$$\frac{d\sigma}{dQ^2 dx} = \frac{2\pi \alpha^2 y^2}{Q^4} \left[2F_1(x) + \frac{1}{2x y^2} F_2(x) \left[(2-y)^2 - y^2 \left(1 + \frac{4x^2 m^2}{Q^2}\right) \right] \right]$$

$\Rightarrow$ Allan – Gross Relation Implication

$$\frac{\sigma_L}{\sigma_T} = \frac{\frac{\nu^2}{Q^2} W_2 - W_1}{W_1} =$$

$$= \frac{\left(1+\frac{\nu^2}{Q^2}\right) W_2 - W_1}{W_1} =$$

$$= \frac{\left(1+\frac{\nu}{2Mx}\right) W_2 - W_1}{W_1} =$$

$$= \frac{\left(1+\frac{\nu}{2Mx}\right)\frac{F_2}{\nu} - \frac{F_1}{M}}{\frac{F_1}{M}} =$$

$$= \frac{\left(1+\frac{\nu}{2Mx}\right)\frac{MF_2}{\nu} - F_1}{F_1}$$

$\nu \to \infty$

x fixed

F_1

$CG \approx F_1$

$$= \frac{\frac{1}{2X} F_2 - F_1}{F_1} = 0$$

$$\frac{\sigma_L}{\sigma_T} \sim 0 \quad \text{fixed } X \quad \nu \to \infty$$

$$\boxed{\frac{1}{\nu} \ll \frac{1}{2MX}} \quad \boxed{\nu \gg 2MX}$$