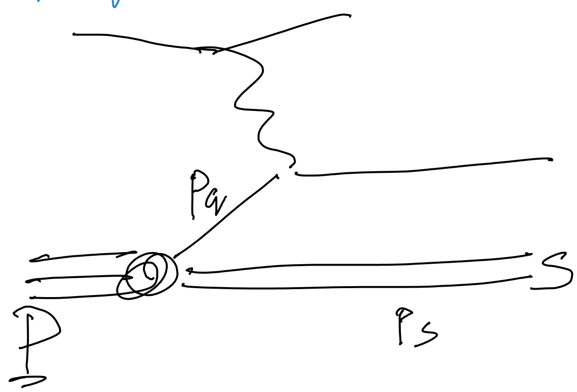
Lecture13

Wednesday, November 15, 2017 12:49 PM

QCD Evolution Equation.

⇒ We discussed a reaction



$$P_q^z = x P_p$$

Introduced $\Psi_p(x, P_{q_i}) = \dfrac{\psi^+_{s_p} U_{s_i}(P_q)\,\Gamma\, U_{s_p}(P)}{2E_q(E_p - E_s - E_q^{on})} \times 2(2\pi)^3$

$$A^\mu = \sum_{s_i} \bar{U}_{s_f}(P_{q_f})\, e_q \gamma^\mu U_{s_i}(P_{q_i}) \frac{\Psi(x, P_{q_i})}{x\,(2\pi)^3}$$

$$M = \underbrace{\bar{u}(k_f)\,\gamma^\mu u(k_i)}_{\text{electron}} \frac{g_{\mu\nu}}{q^2} \underbrace{A^\nu}_{\text{Nucleon}}$$

$$|M|^2 = L^{\mu\nu} H_{\mu\nu} \frac{e^4}{q^4}$$

$$W_{\mu\nu} = \frac{1}{4\pi M_N} H^{\mu\nu} (2\pi)^4 \delta^4(q + P_N - P_{qf} - P_{qs})$$

$$\Rightarrow \frac{d^3 P_{qf}}{\ldots} \quad d^3 P_{qs}$$

$$\frac{\ }{2E_{qf}(2\pi)^3} \quad \frac{\ }{2E_s(2\pi)^3}$$

$$\boxed{\begin{aligned} \bar{H}^{\mu\nu} &= e_q^2 \tfrac{1}{2} \mathrm{Tr}\left(\not{P}_{qf}\gamma^\mu \not{P}_{qi}\gamma^\nu\right) = \\ & e_q^2 \left[4\left(P_{qi}^\mu + \frac{q^\mu}{2}\right)\left(P_{qi}^\nu + \frac{q^\nu}{2}\right) + O\left(g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)\right] \\ & \times \frac{|\psi|^2}{x^2\, 2(2\pi)^3} = e_q^2 \hat{H}^{\mu\nu} \frac{|\psi|^2}{x^2\, 2(2\pi)^3} \end{aligned}}$$

$$\frac{d^3 P_{qf}}{2E_f} = \delta^4(P_{qf}^2 - m^2)\, d^4 P_f$$

$$W_{\mu\nu} = \frac{1}{4\pi M_N^2} \Sigma \bar{H}^{\mu\nu} (2\pi)\, \delta(P_{fq}^2 - m^2) \frac{d^3 P_{qs}}{2E_s(2\pi)^3} \quad //$$

$$P_{fq}^2 - m^2 = (P_{fi} + q)^2 - m_q^2 = 2P_{fi}q - Q^2 =$$

$$= x \cdot 2P_N q - Q^2$$

$$= \frac{1}{2M_N^2} \Sigma \bar{H}^{\mu\nu} \frac{1}{2P_N q} \delta(x - x_{Bj}) \frac{dx_s}{x_s} \frac{d^2 P_{qss}}{2(2\pi)^3}$$

$$= \frac{1}{2M_N} e_q^2 \hat{H}^{\mu\nu} \frac{|\psi(x, P_{qi})|^2}{x^2\, 2(2\pi)^3} \frac{1}{2P_N q} \delta(x - x_{Bj}) \frac{dx_s}{x_s} \frac{d^2 P_{qss}}{2(2\pi)^3}$$

$$= \underbrace{\frac{1}{2M_N^2} \frac{\Sigma \hat{H}^{\mu\nu}}{Q^2}}_{\hat{W}_{\mu\nu}} e_q^2 f_q(x) = //$$

$$|\psi(\ldots)|^2 \delta(1 - x_i - x_s)\, \delta(x - x_{Bj})$$

where $f_q(x) = |\Psi(x, \ldots)|^2 \ldots$

$$\times\, \delta^2(\hat{p}_{q_\perp} + p_{qs_\perp}) \frac{dx_i}{x_i}\frac{d^2 p_{qi_\perp}}{2(2\pi)^2}\frac{dx_s}{x_s}\frac{d^2 p_{qs_\perp}}{2(2\pi)^3}$$

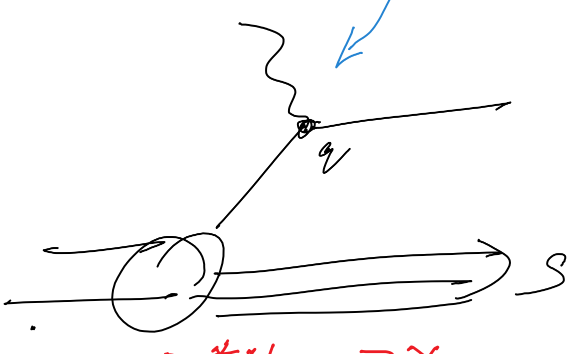
$$W_{\mu\nu} = \sum_q \hat{W}_q^{\mu\nu} f_q(x)$$

$\Rightarrow$ From Lecture 10

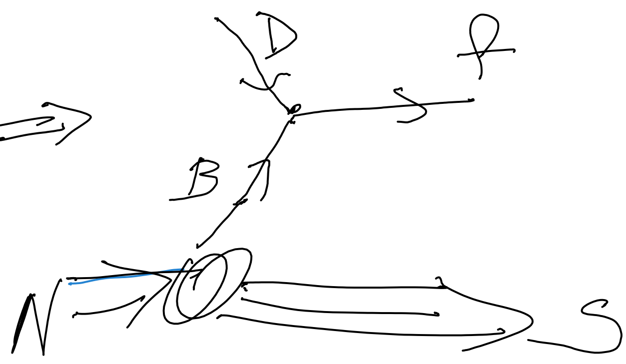
$$\frac{d\sigma^{eN}}{dE'\,d\Omega} = \frac{\alpha^2}{q^4}\frac{E'}{E}\left(L^{\mu\nu}\right)W^{\mu\nu} =$$

$$= \sum_q \frac{\alpha^2}{q^4}\frac{E'}{E} L^{\mu\nu}\hat{W}_q^{\mu\nu} e_q^2 f_q(x) =$$

$$= \sum_q \frac{d\sigma^{eq}}{dE'\,d\Omega} e_q^2 f_q(x)$$



$\Rightarrow$



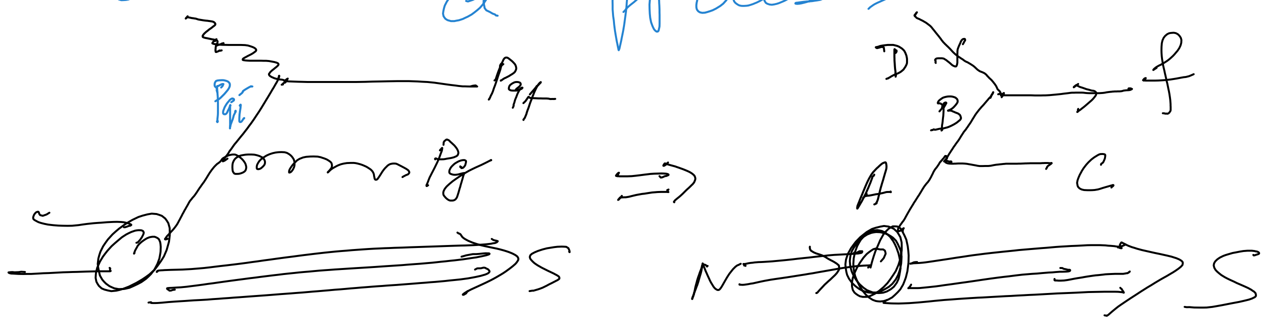
$\gamma^* N \to X$

$DN \to X$

$$\sigma^{DN\to f+S} = \sum_q \sigma^{DB\to f} f_q(x)$$

$f_i S$ $\qquad q'$ $\qquad \boxed{x = \frac{Q^2}{2M\nu}}$

$\Rightarrow$ We now would like to calculate a process in which



$DN \to X$

$$\sum_{f,C,S} \sigma^{D+N\to f+C+S} = \sum_{f\,C} \sum_q \sigma^{A+D\to C+f} f_q(x)$$

$$= \sum_{f\,C} \sum_q \sigma^{D+B\to f} \cdot \sigma^{A\to B+C} f_q(y)\,\frac{dy}{y}$$

$z = \frac{P_B^z}{P_A^z}$ $\qquad y = \frac{P_A^z}{P_N}$

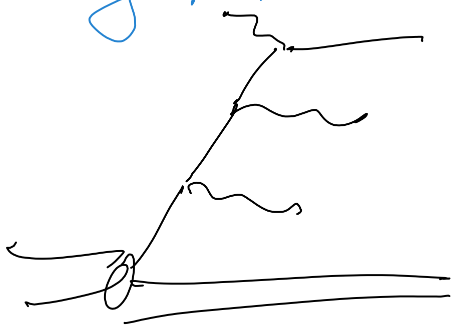
$\delta(yz - x)$

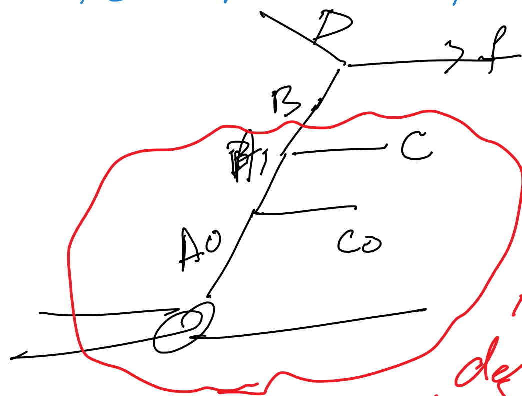
$$\sigma^{DN\to X} = \sum_f \sum_q \sigma^{D+B\to f} \underbrace{\left(f_q(x) + \sigma^{A\to B+c} f_q(y)\,\frac{dy}{y} \right)}_{\tilde{f}(x)}$$

ff

$\tilde{f}(x)$: New distribution

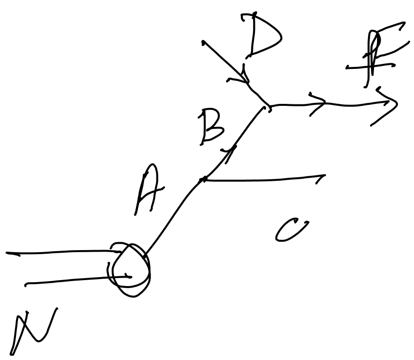
⇒ This scheme can be generalized to more splittings

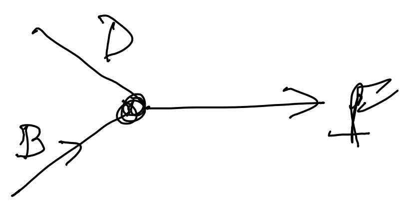




absorbed into the definition of partonic distribution!

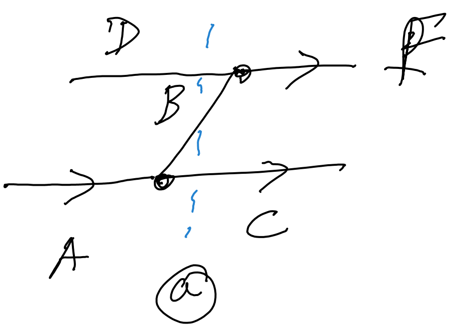
⇒ Calculation of $\sigma^{A+D\to C+f}$ through $\sigma^{B+D\to f}$

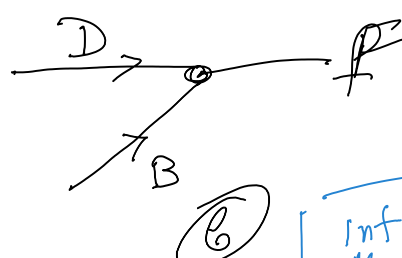




⇒ Considering in $P_N \to \infty$

$P_A \gg P_N \to \infty$





introducing Matrix Element $D \to f$

(a)

$$\begin{vmatrix} \nearrow B \\ V_{D+B\to f} \end{vmatrix}$$

$$M_{A+D\to C+F} \doteq \frac{V_{A\to B+C}\, V_{D+B\to F}}{2E_B\,(E_A - E_B - E_C)}$$

(b) $M_{B+D\to F} = V_{B+D\to F} = V_{D+B\to F}$

$$d\sigma_a = \frac{1}{\underbrace{4\sqrt{P_A P_D}}_{8\,E_A E_D}} \sum_{spin} \frac{|V_{A\to B+C}|^2\,|V_{D+B\to F}|^2}{(2E_B)^2\,(E_B+E_C-E_A)^2}$$

$$\times (2\pi)^4\,\delta^4(K_A+K_D-K_C-K_F)\,\frac{d^3K_C}{(2\pi)^3\,2E_C}\,\frac{d^3K_F}{(2\pi)^3\,(2E_F)}$$

$\Rightarrow$ Ref. Frame $P_A \uparrow\downarrow P_D$ $P_A P_D = 2E_A E_D$

$\Rightarrow$ Assuming $P_B \uparrow\uparrow P_A$ — colinear approximation

$$d\sigma_B = \frac{1}{8E_B E_D}\,|V_{D+B\to F}|^2\,(2\pi)^4\,\delta^4(K_D+K_B-K_F)\,\frac{d^3P_F}{(2\pi)^3\,2E_F}$$

$$d\sigma_a = \frac{E_B}{\ } \frac{|V_{A\to B+C}|^2}{\ } \cdot \frac{d^3K_C}{\ } \cdot d\sigma_B$$

$$E_A \quad (2E_B)^2 (E_B+E_C-E_A) \quad (2\pi)\, 4E_C$$

$$dP_{BA}(z)\, dz$$

Probability of finding partial B in A

$$z = \frac{p_B^z}{p_A}$$

$$d\sigma^{D+A\to C+F} = \boxed{dP_{BA}(z)\, dz}\, d\sigma^{D+B\to F}$$

$$dP_{BA}(z)\, dz = \frac{E_B}{E_A} \frac{|V_{A\to B+C}|^2}{(2E_B)^2 (E_B+E_C-E_A)^2} \frac{d^3 k_C}{(2\pi)^3 (2E_C)}$$

- neglecting masses

$$\boxed{p_A^z \equiv p}$$

$$- K_A = (p_A, p_A^z, 0, 0)$$

$$- K_B = \left(zp + \frac{p_\perp^2}{2zp},\ zp,\ p_\perp\right)$$

$$- K_C = K_A - K_B = \left((1-z)p + \frac{p_\perp^2}{2(1-z)p},\ (1-z)p,\ -p_\perp\right)$$

(1) $$(2E_B)^2 (E_B+E_C-E_A)^2 = \left[2\left(zp + \frac{p_\perp^2}{2zp}\right)\right]^2 \times$$

$$\times \left(p + \frac{p_\perp^2}{\ldots} + \frac{p_\perp^2}{\ldots} - p\right]^2 =$$

$$\wedge\left[\;\; \frac{}{2z p}\;\; \frac{}{z(1-z)p}\;\;\right]$$

$$= 4z^2p^2\left[\frac{P_\perp^2}{2P}\;\frac{}{z(1-z)}\right]^2 = \frac{(P_\perp^2)^2}{(1-z)^2}$$

$$\boxed{(2E_B)^2(E_B+E_C-E_A)^2 = \frac{(P_\perp^2)^2}{(1-z)^2}}$$

(2) $$\boxed{\frac{d^3K_c}{(2\pi)^3(2E_c)}} = \frac{dK_c^z\, K_c^\perp\, dK_{c\perp}\, d\varphi^{\to 2\pi}}{(2\pi)^3\; 2(1-z)P} =$$

$$= \frac{P\; dz\,\frac{1}{2}dP_\perp^2}{4\pi^2\; z(1-z)P} = \boxed{\frac{dz\, dP_\perp^2}{16\pi^2(1-z)}}$$

$$\boxed{dP_{BA}(z)\,dz} = z\sum_{spin}\frac{|V_{A+B\to C}|^2}{(P_\perp^2)^2}(1-z)^2\,\frac{dz\,dP_\perp^2}{16\pi^2(1-z)} =$$

$$= \frac{1}{8\pi^2}\,\frac{z(1-z)}{2}\sum_{spins}\frac{|V_{A+B\to C}|^2}{(P_\perp^2)^2}\,dz\,dP_\perp^2 =$$

$$= \frac{1}{\;}\;\frac{z(1-z)}{\;}\sum_{spin}\frac{|V_{A+B\to C}|^2}{\;}\,dz\,d\ln P_\perp^2$$

$8\pi^2 \quad 2 \quad " \quad (P_\perp)^2$

$\Rightarrow$ Considering Back the Equations (+4)
One obtains

$$\tilde{f}(x) = f(x) + \int dP_{BA}(z)\, dz \cdot f(y) \frac{dy}{y}$$

(++4)

$$y = \frac{K_A^2}{P_N} \qquad z = \frac{K_B^2}{K_A}$$

- for electroproduction reaction $x = \frac{Q^2}{2P_N \cdot q}$

$$-(K_D + K_B)^2 = K_P^2 = M_f^2 = (q + K_B)^2 = M_f^2$$

$$-Q^2 + 2qK_B + \underbrace{M_B^2 = M_f^2}_{\text{"equal"}}$$

$$-Q^2 + 2qK_B = 0$$

$$-Q^2 + 2qK_A \cdot z = 0 \qquad \boxed{z} = \frac{Q^2}{2qK_A} = \frac{Q^2}{2qP_N \cdot y} = \boxed{\frac{x}{y}}$$

$$\boxed{y = \frac{K_A^2}{P_N}}$$

(***) can be expressed as

$$\hat{f}(x) = \int_0^1 dy \int_0^1 dz\, \delta(zy - x)\, f(y) \left[\underset{a}{\delta(z-1)} + \underset{b}{dP_{BA}(z)} \right]$$

$\Rightarrow$ checking (a) $\int_0^1 dy \int_0^1 dz\, \delta(zy-x) f(y) \delta(z-1) =$

$$= \int_0^1 dy\, \delta(y-x) f(y) = f(x)$$

(b) $$= \int_0^1 \frac{dy}{y} f(y)\, dP_{BA}\left(\frac{x}{y}\right) \;//$$