Lecture3

Monday, September 25, 2017   2:30 PM

Photon Propagator

⇒ Consider Now a process with initial ee and final e-e

$$\frac{[-ie^2]}{2} T \int \bar{\psi}(x_2) \not{A}(x_2) \psi(x_2) dx_2 \int \bar{\psi}(x_1) \not{A}(x_1) \psi(x_1) dx_1$$

$t_2 > t_1$

$$\frac{[-ie^2]}{2} \int \bar{\psi}(x_2) \not{A}(x_2) \psi(x_2) dx_2 \int \bar{\psi}(x_1) \not{A}(x_1) \psi(x_1) dx_1 \; \theta(x_2 - x_1)$$

⇒ We consider

$$D_{\mu\nu}(x_2 - x_1) = A^{\mu}(x_2) A^{\nu +}(x_1) \theta(x_2 - x_1)$$

which equation the photon propagator satisfies?

⇒ for electrons $(i\gamma^{\mu}\partial_{\mu} - m) G(x_2, x_1) = i\delta^4(x_2 - x_1)$

because $(i\gamma^{\mu}\partial_{\mu} - m)\psi_e = 0$

For $A^{\mu}(x)$ one has

$$\partial_{\mu}^2 A^{\nu} - \partial_{\nu}(\partial_{\mu} A_{\mu}) = 0$$

$$\partial_{\lambda}^2 A^{\mu} - \partial_{\mu}(\partial_{\sigma} A_{\sigma}) = 0$$

Follows

$$[g_{\mu\sigma}\partial_{\lambda}^2 - \partial_{\mu}\partial_{\sigma}] D_{\sigma\nu}(x) = i g_{\mu\nu}\delta^4(x)$$

$D_{\sigma\nu} \to -i D_{\sigma\nu}$

$$D_{\mu\nu}(x_2 - x_1) = \int e^{-ik(x_2 - x_1)} D_{\mu\nu}(k) \frac{d^4k}{(2\pi)^4}$$

$$\left(k^2 g_{\mu\sigma} - k_\mu k_\sigma\right) D_{\sigma\nu}(k) = g_{\mu\nu} \qquad (1)$$

Need to solve for $D_{\sigma\nu}(k)$

$\Rightarrow$ simple inversion does not work since it has singularity $D_{\sigma\nu} = \dfrac{g_{\mu\nu}}{k^2 g_{\mu\sigma} - k_\nu k_\nu} \to 0$

$\Rightarrow$ looking for the solution in the form

$$D_{\sigma\nu}(k) = A(k^2) g'_{\sigma\nu} + B(k^2) \frac{k_\sigma k_\nu}{k^2} \qquad (2)$$

$$g'_{\sigma\nu} = g_{\sigma\nu} - \frac{k_\sigma k_\nu}{k^2}$$

$g'_{\sigma\nu} k_\sigma k_\nu = 0$

$g'_{\sigma\nu} \perp k_\sigma k_\nu$

Also

$g'_{\sigma\lambda} g'_{\lambda\nu} = g'_{\sigma\nu}$

$g_{\sigma\sigma} = 3$

$\Rightarrow$ insert (2) into (1)

$$\left(k^2 g'_{\mu\sigma} + k_\mu k_\sigma - k_\mu k_\sigma\right)\left(A g'_{\sigma\nu} + B \frac{k_\sigma k_\nu}{k^2}\right) = g'_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}$$

$$k^2 A g'_{\mu\sigma} g'_{\sigma\nu} + \underbrace{B g'_{\mu\sigma} k_\sigma k_\nu}_{=0} = g'_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}$$

$$k^2 A g'_{\mu\nu} = g'_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}$$

— B-term is dropped out ⇒ solution is not unique

— One can not find solution for $D_{\sigma\nu}(k)$

⇒ To solve this issue one adds to the Lagrange Density a Noninvariant term

$$\Delta\mathcal{L} = -\frac{1}{2\xi}(\partial_\mu A_\mu)^2$$

and then take $\xi \to \infty$

⇒ This term results in the following Equation of Motion

$$\mathcal{L}_{mod} = \mathcal{L} + \Delta\mathcal{L}$$

$$\to \partial_\mu F_{\mu\nu} + \frac{1}{\xi}\partial_\nu(\partial_\mu A_\mu) = J_\nu(x)$$

⇒ For Green Function the result is

$$\boxed{\left(K^2 g'_{\mu\sigma} + \frac{K_\mu K_\sigma}{\xi}\right) D_{\sigma\nu}(k) = g_{\mu\nu} + \frac{K_\mu K_\nu}{K^2}}$$

⇒ Now we can find the solution

$$\boxed{D_{\sigma\nu}(k) = A g'_{\sigma\nu} + \frac{B K_\sigma K_\nu}{K^2}}$$

$$\left(K^2 g'_{\mu\sigma} + \frac{K_\mu K_\sigma}{\xi}\right)\left(A g'_{\sigma\nu} + \frac{B K_\sigma K_\nu}{K^2}\right) = g'_{\mu\nu} + \frac{K_\mu K_\nu}{K^2}$$

$$\Rightarrow \quad K^2 A g'_{\mu\nu} + B\frac{K_\mu K_\nu}{\xi} = g'_{\mu\nu} + \frac{K_\mu K_\nu}{K^2}$$

$$\boxed{A = \frac{1}{K^2} \qquad B = \frac{\xi}{K^2}}$$

$$\boxed{D_{\sigma\nu}(k) = g'_{\sigma\nu} + \frac{\xi K_\mu K_\nu}{K^2} = g_{\sigma\nu} - (1-\xi)\frac{K_\sigma K_\nu}{K^2}}$$

⇒ Then we take $\xi \to \infty$

⇒ However the result should not depend on ξ, since it is proportional to the longitudinal part of the propagator

$\Rightarrow$ Back to Eq. of Motion

$$\partial_\lambda \times \left(\partial_\mu F_{\mu\nu} + \frac{1}{\xi} \partial_\nu (\partial_\mu A_\mu) = j_\nu \right)$$

$$\frac{1}{\xi} \partial_\nu^2 (\partial_\mu A_\mu) = \partial_\nu j_\nu = 0$$

$\hookrightarrow$ current conservation

$$\partial_\nu^2 \eta(x) = 0$$

where $\boxed{\eta(x) = \partial_\mu A_\mu}$ – Longitudinal Photons

$\hookrightarrow$ These photons do not interact.

$\Rightarrow$ $\Delta\mathcal{L}$ is a field of longitudinal photons which does not interact

$\Rightarrow$ adding $\Delta\mathcal{L}$ – procedure is called gauge Fixing

$$\Rightarrow \Delta\mathcal{L} = -\frac{1}{2\xi}(\partial_\mu A_\mu)^2 \text{ – Lorentz Gauge}$$

$\Rightarrow$ Other Gauges

Class of Axial Gauges

Defined through some vector b_μ

in a given reference frame

In a given reference frame

$$\Delta \mathcal{L} = + \frac{1}{2\xi_1 b_\lambda^2} \left(b_\mu A_\mu(x) \right) \partial_\sigma^2 \left(b_\nu A_\nu(x) \right)$$

$$\delta(\Delta \mathcal{L}) = + \frac{1}{2\xi_1 b_\lambda^2} \left(b_\mu \, \delta A_\mu(x) \right) \partial_\sigma^2 \left(b_\nu A_\nu \right)$$

$$\partial_\mu F_{\mu\nu} - \frac{1}{\xi_1 b_\lambda^2} b_\nu \partial_\sigma^2 \left(b_\nu A_\nu \right) = J_\nu(x)$$

Green Function

$$\left[k^2 g'_{\mu\sigma} - \frac{k^2 b_\mu b_\sigma}{\xi_1 b_\lambda^2} \right] D_{\sigma\nu}(k) = g_{\mu\nu}(k)$$

Looking for solution

$$D_{\sigma\nu} = A g_{\sigma\nu} + \frac{k_\sigma b_\nu + b_\sigma k_\nu}{k_\lambda b_\lambda} B + \frac{b_\nu^2 k_\sigma k_\nu}{(b_\lambda k_\lambda)^2} C$$

$$A = -B = \frac{1}{k^2} \qquad C = \frac{(1 - \xi_1)}{k^2}$$

$$D_{\sigma\nu} = \frac{d_{\sigma\nu}}{k^2 + i\varepsilon}$$

$$d_{\sigma\nu} = g_{\sigma\nu} - \frac{k_\sigma b_\nu + b_\sigma k_\nu}{b_\lambda k_\lambda} + \frac{b_\lambda^2 k_\sigma k_\nu}{(b_\lambda k_\lambda)^2} (1 - \xi_1)$$

⇒ In this case $\tilde{\eta}(x) =$ [illegible] photons do not interact $\partial_\sigma^2 \tilde{\eta} = 0$

⇒ $\xi_1 = 1$ — Planar Gauge

⇒ $\xi_1 = 0$; $e_\mu = (1, \vec{0})$ — radiation Gauge

$e_\mu = (0, \vec{k})$ — Coulomb Gauge

⇒ Consider the amplitude