

A SAS Program for Bootstrap Estimation of the Generalized Linear Mediation Model

Introduction

Statistical mediation analysis allows a researcher to examine the causal path from a predictor (X) through a mediator (M) to an outcome (Y). The product of the a regression coefficient (from the regression of M on X) and the b regression coefficient (from the regression of Y on M controlling for X), $a \times b$, is a quantitative measure of the mediated effect.

There are several methods available to calculate confidence intervals and evaluate the significance of the mediated effect. MacKinnon et al. (2004) evaluated the statistical power and type I error rates of several methods to test the mediated effect; the non-parametric bootstrap test was the best performing of the methods tested. The bootstrap test also has the advantage of easily expand beyond the single-mediator model with continuous and conditionally normally-distributed M and Y.

Categorical outcomes have been shown to complicate the estimation of mediation effects (MacKinnon & Dwyer, 1993; Coxe & MacKinnon, 2010). Discrete or categorical variables typically violate the assumptions for an outcome in the standard linear regression model. Regression models from the broader generalized linear model (GLiM) family, such as logistic regression and Poisson regression, are the preferred method for categorical outcomes. Standard methods for testing the mediated effect become either inappropriate or algebraically unsolvable because the sampling distribution for regression coefficients in GLiMs does not follow the same distribution as regression coefficients in linear regression.

This poster describes an adaptable bootstrap method for mediation models with GLiM components that is implemented in a SAS macro program; this addresses both the problems of estimating mediation for categorical outcomes and the success of bootstrap methods. This macro uses the non-parametric bootstrap method to provide confidence intervals when the mediator and/or the outcome are discrete (e.g., binary, ordered categories, or counts). Both the percentile and bias-corrected bootstrap (Efron & Tibshirani, 1994) are presented.

David P. MacKinnon and Stefany Coxe
Arizona State University



Method

Bootstrapping is a resampling method used to construct confidence intervals and assess significance. Repeated samples of size n are taken (with replacement) from the observed data; these are referred to as "bootstrap samples." For each bootstrap sample, the mediated effect ($a \times b$) is calculated; the values of the mediated effect from the bootstrap samples are used to create an empirical, observed distribution for the mediated effect. Confidence intervals and statistical significance can be calculated based on this empirical distribution rather than a theoretical distribution. The theoretical distribution of the product of coefficients is difficult to calculate (MacKinnon, et al., 2007); the distribution is further complicated if M or Y (or both) are categorical.

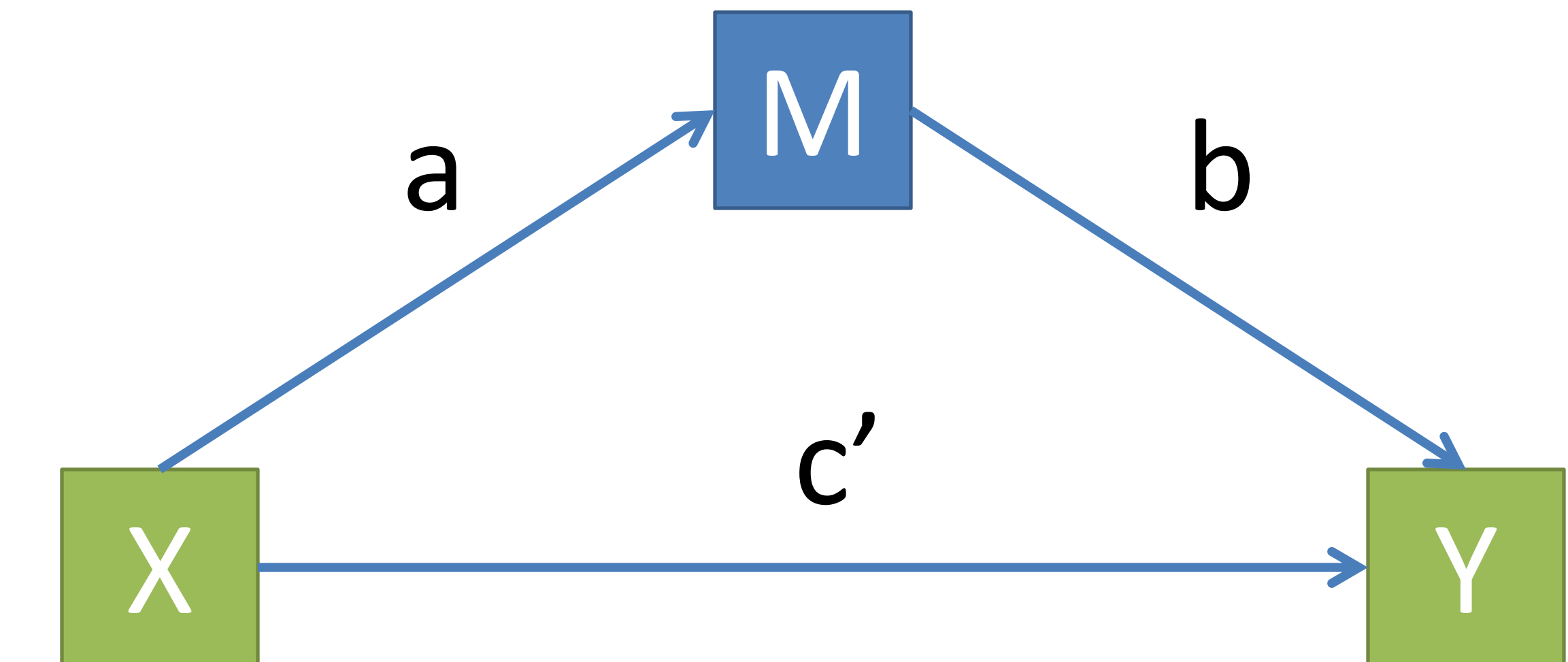
This SAS macro calculates the mediated effect, the percentile bootstrap confidence interval for the mediated effect, and the bias-corrected bootstrap confidence interval for the mediated effect.

The choice of analysis model for M and Y is typically based on the variable type; models include linear regression (OLS) for continuous outcomes, logistic regression and probit regression for binary variables, ordinal logistic regression for ordered categorical variables, and Poisson regression, overdispersed Poisson regression, and negative binomial regression for count variables. There are also options for multiple X variables (for example, 2 dummy coded variables for an intervention with 3 conditions), covariates for both M and Y, and varying numbers of bootstrap samples.

Two examples are provided here to demonstrate the confidence intervals obtained by the macro. The figures below show the empirical bootstrap distribution of the mediated effect for these two example data sets. The red arrow indicates the estimate of the mediated effect. The blue bracket shows the percentile bootstrap confidence interval; the green bracket shows the bias-corrected bootstrap confidence interval.

Conclusions

Categorical variables are increasingly used as outcomes in the behavioral sciences; for example, symptom counts and binary diagnoses are common outcomes in prevention research. This macro provides a simple and effective way of calculating confidence intervals and determining significance of the mediated effect for these situations in which M and/or Y are categorical and must be analyzed with the GLiM family of regression models.



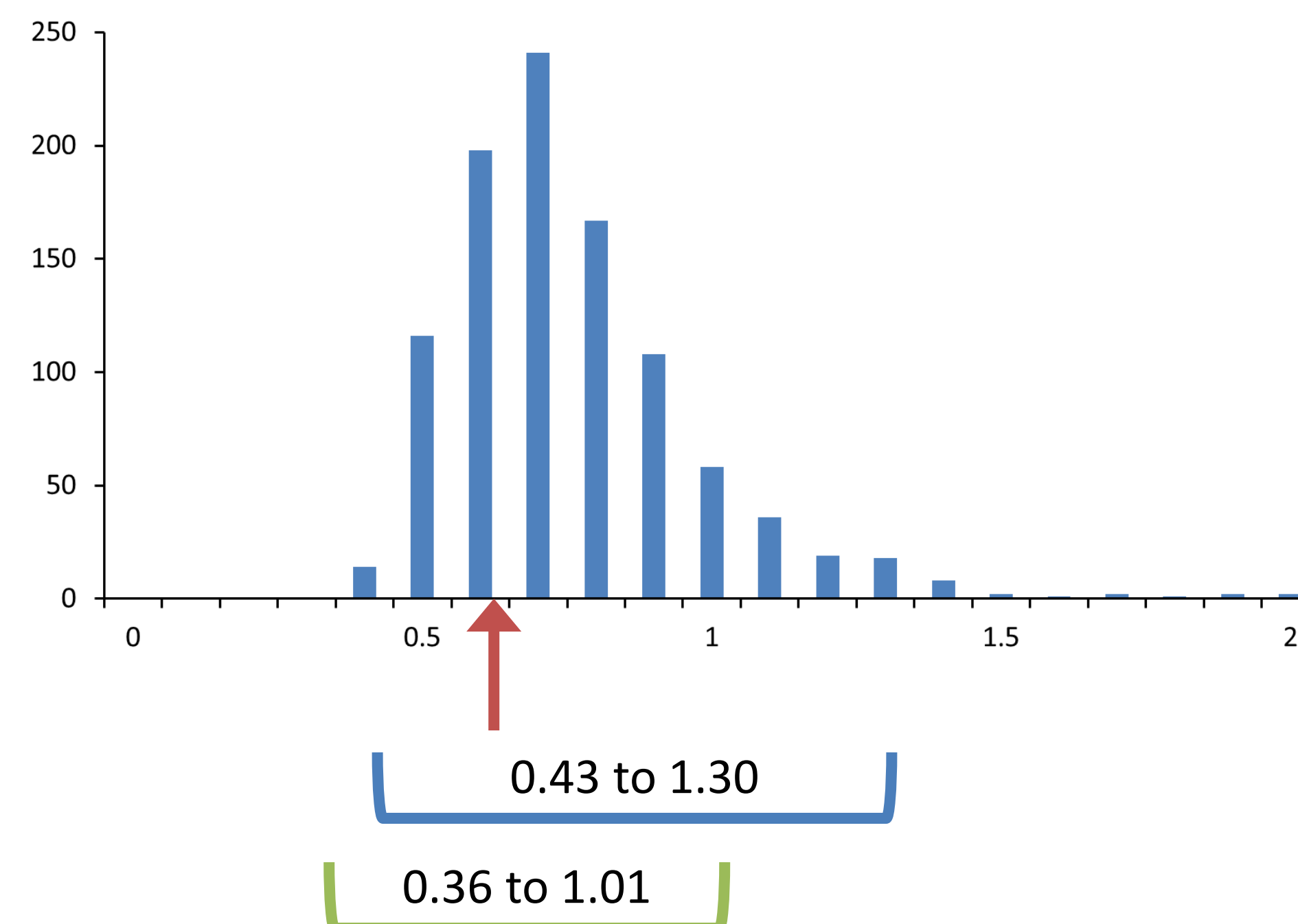
Example 1

Simulated data
N = 250
The data were generated using the relationships:
 $\hat{M} = 0.5X$
 $\hat{Y} = e^{0.5X + 0.5M}$
The true $a \times b$ mediated effect in the simulated data was 0.82

X is continuous and normally distributed
M is continuous and normally distributed
Y is a count following a Poisson distribution

M analyzed with OLS regression
Y analyzed with overdispersed Poisson regression

1000 bootstrap samples



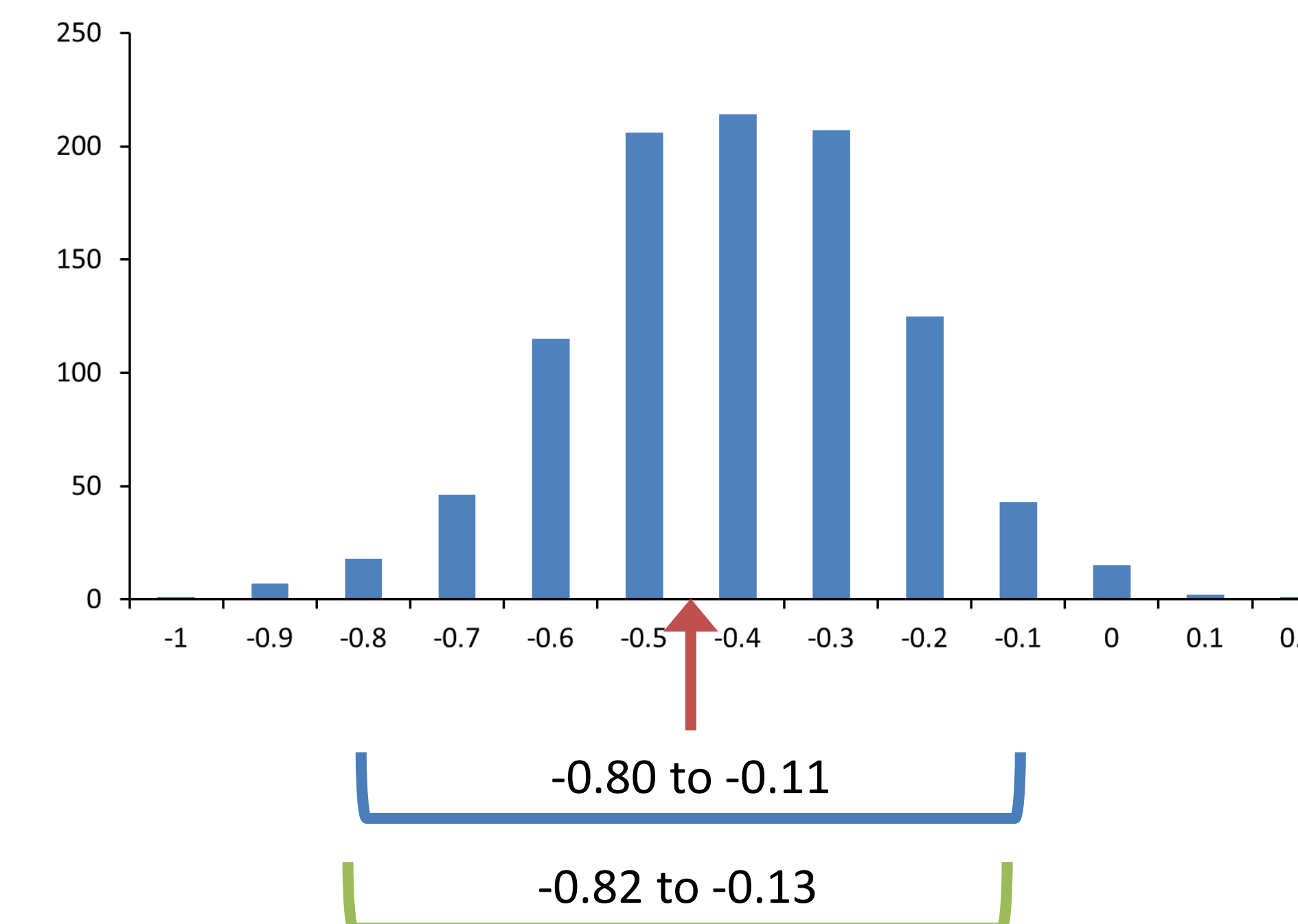
Example 2

Data from the Midwestern Prevention Project, presented in MacKinnon et al. (2007, *Clinical Trials*)
N = 864

X is binary (intervention program)
M is an ordered category (intention to use cigarettes: yes/probably/don't think so/no)
Y is binary (cigarette use: yes/no)

M analyzed with ordinal logistic regression
Y analyzed with binary logistic regression

1000 bootstrap samples



References

- Coxe, S. and MacKinnon, D. P. (2010). Mediation analysis of Poisson-distributed count outcomes. *Multivariate Behavioral Research, 45*. (Abstract).
- Efron, B., & Tibshirani, R. J. (1993). *An introduction to the bootstrap*. New York: Chapman & Hall.
- MacKinnon, D. P. & Dwyer, J. H. (1993). Estimating mediated effects in prevention studies, *Evaluation Review, 17*, 144-158.
- MacKinnon, D. P., Fritz, M. S., Williams, J., & Lockwood, C. M. (2007). Distribution of the product confidence limits for the indirect effect: Program PRODCLIN. *Behavior Research Methods, 39*, 384-389.
- MacKinnon, D. P., Lockwood, C. M., Brown, C.H., & Hoffman, J. M. (2007). The intermediate endpoint effect in logistic and probit regression. *Clinical Trials, 4*, 499 - 513.
- MacKinnon, D. P., Lockwood C. M., & Williams, J. (2004). Confidence limits for the indirect effect: Distribution of the product and resampling methods. *Multivariate Behavioral Research, 39*, 99-128.