Example of vector calculus

```
In[2]:= er = {Sin[theta] Cos[phi], Sin[theta] Sin[phi], Cos[theta]};
        ephi = {-Sin[phi], Cos[phi], 0};
        etheta = {Cos[theta] Cos[phi], Cos[theta] Sin[phi], -Sin[theta]};
```

```
Simplify[Dot[etheta, ephi]]
```

```
0
```

projection of del r

```
Simplify[Dot[Dt[r * er], er]]
Simplify[Dot[Dt[r * er], etheta]]
Simplify[Dot[Dt[r * er], ephi]]
```

```
Dt[r]

r Dt[theta]

r Dt[phi] Sin[theta]
```

gradient of scaler function first change into cartesian, after gradient project onto spherical unit vectors

```
fun = r^2 Cos[theta] Sin[phi]
```

r^2 Cos[theta] Sin[phi]

```
newfun =
 Simplify[fun /. r -> Sqrt[x^2 + y^2 + z^2] /. theta -> ArcCos[z / Sqrt[x^2 + y^2 + z^2]] /.
   phi -> ArcTan[y / x]]
```

$$\frac{y\, z \sqrt{x^2 + y^2 + z^2}}{x \sqrt{1 + \frac{y^2}{x^2}}}$$

```
grad = {D[newfun, x], D[newfun, y], D[newfun, z]}
```

$$\left\{\frac{y\, z}{\sqrt{1 + \frac{y^2}{x^2}} \sqrt{x^2 + y^2 + z^2}} + \frac{y^3 z \sqrt{x^2 + y^2 + z^2}}{x^4 \left(1 + \frac{y^2}{x^2}\right)^{3/2}} - \frac{y\, z \sqrt{x^2 + y^2 + z^2}}{x^2 \sqrt{1 + \frac{y^2}{x^2}}},\right.$$
$$\frac{y^2 z}{x \sqrt{1 + \frac{y^2}{x^2}} \sqrt{x^2 + y^2 + z^2}} - \frac{y^2 z \sqrt{x^2 + y^2 + z^2}}{x^3 \left(1 + \frac{y^2}{x^2}\right)^{3/2}} + \frac{z \sqrt{x^2 + y^2 + z^2}}{x \sqrt{1 + \frac{y^2}{x^2}}},$$
$$\left.\frac{y\, z^2}{x \sqrt{1 + \frac{y^2}{x^2}} \sqrt{x^2 + y^2 + z^2}} + \frac{y \sqrt{x^2 + y^2 + z^2}}{x \sqrt{1 + \frac{y^2}{x^2}}}\right\}$$

```
newgrad = Simplify[
  grad /. x -> r Sin[theta] Cos[phi] /. y -> r Sin[theta] Sin[phi] /. z -> r Cos[theta]]
```

$$\left\{-\frac{\sqrt{r^2}\ \mathrm{Cos}[\mathrm{theta}]^2\ \mathrm{Cot}[\mathrm{theta}]\ \mathrm{Sin}[\mathrm{phi}]}{\sqrt{\mathrm{Sec}[\mathrm{phi}]^2}},\ \frac{1}{8\sqrt{\mathrm{Sec}[\mathrm{phi}]^2}}\left(\sqrt{r^2}\ (6+2\,\mathrm{Cos}[2\,\mathrm{phi}]+\mathrm{Cos}[2\,(\mathrm{phi}-\mathrm{theta})]-2\,\mathrm{Cos}[2\,\mathrm{theta}]+\mathrm{Cos}[2\,(\mathrm{phi}+\mathrm{theta})])\ \mathrm{Cot}[\mathrm{theta}]\ \mathrm{Sec}[\mathrm{phi}]\right),\ \frac{1}{4}\sqrt{r^2}\ (3+\mathrm{Cos}[2\,\mathrm{theta}])\sqrt{\mathrm{Sec}[\mathrm{phi}]^2}\ \mathrm{Sin}[2\,\mathrm{phi}]\right\}$$

```
Simplify[Dot[newgrad, er]]
Simplify[Dot[newgrad, etheta]]
Simplify[Dot[newgrad, ephi]]
```

$$\frac{2\sqrt{r^2}\ \mathrm{Cos}[\mathrm{theta}]\ \mathrm{Tan}[\mathrm{phi}]}{\sqrt{\mathrm{Sec}[\mathrm{phi}]^2}}$$

$$-\frac{\sqrt{r^2}\ \mathrm{Sin}[\mathrm{theta}]\ \mathrm{Tan}[\mathrm{phi}]}{\sqrt{\mathrm{Sec}[\mathrm{phi}]^2}}$$

$$\frac{\sqrt{r^2}\ \mathrm{Cot}[\mathrm{theta}]}{\sqrt{\mathrm{Sec}[\mathrm{phi}]^2}}$$

2r Cos[theta]Sin[phi]; -r Sin[theta]Sin[phi]; r Cot[theta]Cos[phi]; Next, directly work out the gradient in spherical coordinates!

```
gradx = D[Sqrt[x^2 + y^2 + z^2], x] * partr +
  D[ArcTan[y / x], x] * partphi + D[ArcCos[z / Sqrt[x^2 + y^2 + z^2]], x] * parttheta;
grady = D[Sqrt[x^2 + y^2 + z^2], y] * partr +
  D[ArcTan[y / x], y] * partphi + D[ArcCos[z / Sqrt[x^2 + y^2 + z^2]], y] * parttheta;
gradz = D[Sqrt[x^2 + y^2 + z^2], z] * partr + D[ArcTan[y / x], z] * partphi +
  D[ArcCos[z / Sqrt[x^2 + y^2 + z^2]], z] * parttheta;
```

```
grad = {gradx, grady, gradz}
```

$$\left\{-\frac{\mathrm{partphi}\ y}{x^2\left(1+\frac{y^2}{x^2}\right)}+\frac{\mathrm{partr}\ x}{\sqrt{x^2+y^2+z^2}}+\frac{\mathrm{parttheta}\ x\ z}{(x^2+y^2+z^2)^{3/2}\sqrt{1-\frac{z^2}{x^2+y^2+z^2}}},\ \frac{\mathrm{partphi}}{x\left(1+\frac{y^2}{x^2}\right)}+\frac{\mathrm{partr}\ y}{\sqrt{x^2+y^2+z^2}}+\frac{\mathrm{parttheta}\ y\ z}{(x^2+y^2+z^2)^{3/2}\sqrt{1-\frac{z^2}{x^2+y^2+z^2}}},\ \frac{\mathrm{partr}\ z}{\sqrt{x^2+y^2+z^2}}-\frac{\mathrm{parttheta}\left(-\frac{z^2}{(x^2+y^2+z^2)^{3/2}}+\frac{1}{\sqrt{x^2+y^2+z^2}}\right)}{\sqrt{1-\frac{z^2}{x^2+y^2+z^2}}}\right\}$$

```
gradnew = Simplify[
  grad /. x -> r Sin[theta] Cos[phi] /. y -> r Sin[theta] Sin[phi] /. z -> r Cos[theta]]
```

$$\left\{\frac{1}{r^2}\left(-\text{partphi}\,r\,\text{Csc}[\text{theta}]\,\text{Sin}[\text{phi}]+\sqrt{r^2}\,\text{Cos}[\text{phi}]\left(\text{partr}\,r\,\text{Sin}[\text{theta}]+\text{parttheta}\,\text{Cot}[\text{theta}]\sqrt{\text{Sin}[\text{theta}]^2}\right)\right),\right.$$
$$\frac{1}{r^2}\left(\text{Cos}[\text{phi}]\left(\text{partphi}\,r\,\text{Csc}[\text{theta}]+\sqrt{r^2}\left(\text{partr}\,r\,\text{Sin}[\text{theta}]+\text{parttheta}\,\text{Cot}[\text{theta}]\sqrt{\text{Sin}[\text{theta}]^2}\right)\text{Tan}[\text{phi}]\right)\right),$$
$$\left.\frac{\text{partr}\,r\,\text{Cos}[\text{theta}]-\text{parttheta}\sqrt{\text{Sin}[\text{theta}]^2}}{\sqrt{r^2}}\right\}$$

```
Simplify[Dot[gradnew, er]]
Simplify[Dot[gradnew, etheta]]
Simplify[Dot[gradnew, ephi]]
```

$$\frac{\text{partr}\sqrt{r^2}}{r}$$

$$\frac{\text{parttheta}\,\text{Csc}[\text{theta}]\sqrt{\text{Sin}[\text{theta}]^2}}{\sqrt{r^2}}$$

$$\frac{\text{partphi}\,\text{Csc}[\text{theta}]}{r}$$

partr; 1/r parttheta; 1/(r Sin[theta]) partphi; apply to fun=r^2Cos[theta]Sin[phi] works!

```
In[36]:= Simplify[Dot[etheta, D[er, theta]] / r]
         Simplify[Dot[etheta, D[etheta, theta]] / r]
         Simplify[Dot[etheta, D[ephi, theta]] / r]
         Simplify[Dot[ephi, D[er, phi]] / r / Sin[theta]]
         Simplify[Dot[ephi, D[etheta, phi]] / r / Sin[theta]]
         Simplify[Dot[ephi, D[ephi, phi]] / r / Sin[theta]]
```

Out[36]= $\frac{1}{r}$

Out[37]= 0

Out[38]= 0

Out[39]= $\frac{1}{r}$

Out[40]= $\frac{\text{Cot}[\text{theta}]}{r}$

Out[41]= 0

partr A_r+ 1/r parttheta A_θ+ 1/(r Sin[theta]) partphi A_ϕ + 2/r A_r + Cot[theta]/r A_θ
Nabla Operator = partr^2 + 1/r^2 parttheta^2 + 1/(r Sin[theta])^2 partphi^2 + 2/r partr + Cot[theta]/r^{2}parttheta