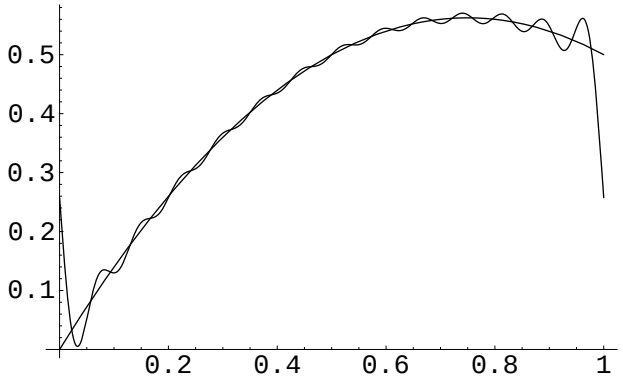
example of calculation of Fourier series expansion for function x(1.5-x) between (0,1). First, just repeat the function in (0,1).

```
f[x_] := x (1.5 - x);
b0 = Integrate[f[x], {x, 0, 1}];

nmax = 13;
afun = Table[Sin[2 π n x], {n, 1, nmax}];
a = Table[2 Integrate[Sin[2 π n x] f[x], {x, 0, 1}], {n, 1, nmax}];
bfun = Table[Cos[2 π n x], {n, 1, nmax}];
b = Table[2 Integrate[Cos[2 π n x] f[x], {x, 0, 1}], {n, 1, nmax}];
ffun[x_] := b0 + Dot[b, bfun] + Dot[a, afun];

Plot[{f[x], ffun[x]}, {x, 0, 1}]
```



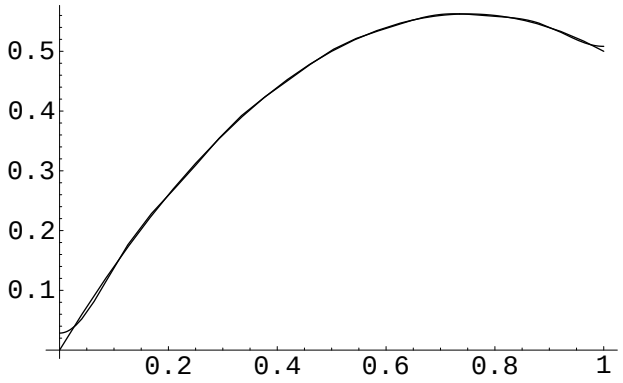

```
- Graphics -
```

Now double the range and do an even (cosine) or odd (sine) extension of the function. Notice the even expansion is superior

```
b0 = 1 * Integrate[f[x], {x, 0, 1}];

nmax = 10;
afun = Table[Sin[π n x], {n, 1, nmax}];
a = Table[ 2 * Integrate[Sin[π n x] f[x], {x, 0, 1}], {n, 1, nmax}];
bfun = Table[Cos[π n x], {n, 1, nmax}];
b = Table[2 * Integrate[Cos[π n x] f[x], {x, 0, 1}], {n, 1, nmax}];
ffun[x_] := b0 + Dot[b, bfun];
```

```
Plot[{f[x], ffun[x]}, {x, 0, 1}]
```




```
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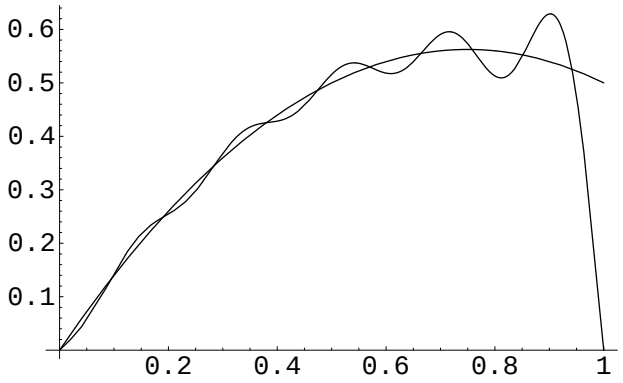
```
ffun[x_] := Dot[a, afun];
```

```
Plot[{f[x], ffun[x]}, {x, 0, 1}, PlotRange -> All]
```




```
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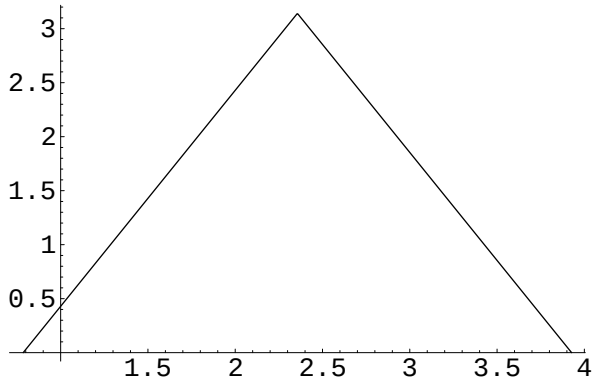
```

an interesting function and its Fourier series

```
Plot[ArcCos[Sin[2 x]], {x, Pi / 4, 5 Pi / 4}]
```




```
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```

```
b0 = 1 / Pi * Integrate[ArcCos[Sin[2 x]], {x, 0, Pi}]
```

$$\frac{\pi}{2}$$

```
bn = 2 / Pi * Integrate[ArcCos[Sin[2 x]] * Cos[2 n x], {x, -Pi / 2, Pi / 2}]
```

$$\frac{\text{Sin}[n\,\pi]}{n}$$

```
an = 2 / Pi * Integrate[ArcCos[Sin[2 x]] * Sin[2 n x], {x, -Pi / 2, Pi / 2}]
```

$$\frac{2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \text{ArcCos}[\text{Sin}[2\,x]]\,\text{Sin}[2\,n\,x]\,dx}{\pi}$$

refused to do integration because derivative ArcCos[Sin[2x]] not continuous, integrate by parts

```
D[ArcCos[Sin[2 x]], x]
```

$$-\frac{2\,\text{Cos}[2\,x]}{\sqrt{1-\text{Sin}[2\,x]^2}}$$

```
-1 / (2 n) * (ArcCos[Sin[2 3 Pi / 4]] Cos[2 n 3 Pi / 4] - ArcCos[Sin[2 Pi / 4]] Cos[2 n Pi / 4]) +
 2 / (2 n) * Integrate[Cos[2 n x], {x, Pi / 4, 3 Pi / 4}]
```

$$-\frac{\pi\,\text{Cos}\left[\frac{3\,n\,\pi}{2}\right]}{2\,n}+\frac{-\frac{\text{Sin}\left[\frac{n\,\pi}{2}\right]}{2\,n}+\frac{\text{Sin}\left[\frac{3\,n\,\pi}{2}\right]}{2\,n}}{n}$$

```
-1 / (2 n) * (ArcCos[Sin[2 5 Pi / 4]] Cos[2 n 5 Pi / 4] - ArcCos[Sin[2 3 Pi / 4]] Cos[2 n 3 Pi / 4]) -
 2 / (2 n) * Integrate[Cos[2 n x], {x, 3 Pi / 4, 5 Pi / 4}]
```

$$\frac{\pi\,\text{Cos}\left[\frac{3\,n\,\pi}{2}\right]}{2\,n}-\frac{-\frac{\text{Sin}\left[\frac{3\,n\,\pi}{2}\right]}{2\,n}+\frac{\text{Sin}\left[\frac{5\,n\,\pi}{2}\right]}{2\,n}}{n}$$

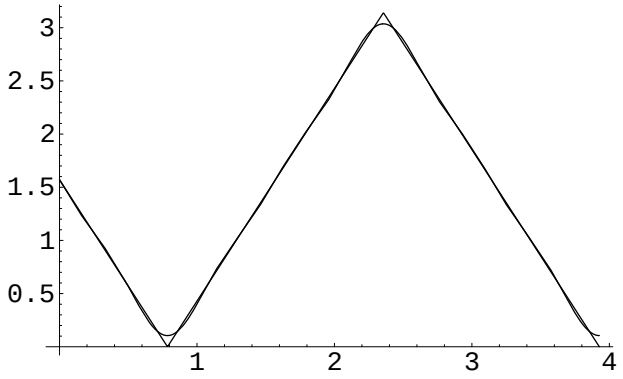
```
Simplify[%19 + %20]
```

$$\frac{2\,(1+2\,\text{Cos}[n\,\pi])\,\text{Sin}\left[\frac{n\,\pi}{2}\right]^3}{n^2}$$

```
fun = b0 + 2 / Pi * Sum[%28 Cos[2 n x] + %21 Sin[2 n x], {n, 1, 5}]
```

$$\frac{\pi}{2}+\frac{2\,\left(-2\,\text{Sin}[2\,x]+\frac{2}{9}\,\text{Sin}[6\,x]-\frac{2}{25}\,\text{Sin}[10\,x]\right)}{\pi}$$

```
Plot[{fun, ArcCos[Sin[2 x]]}, {x, 0, 5 Pi / 4}]
```




```
- Graphics -
```