Example of solve linear equation Mx=b, where M is a 3x3 matrix

```
b = {1, 2, 2};
```

```
MatrixForm[m = Inverse[{{1, 2, 1}, {2, 7, 4}, {1, 8, 7}}]]
```

$$\begin{pmatrix} \frac{17}{6} & -1 & \frac{1}{6} \\ -\frac{5}{3} & 1 & -\frac{1}{3} \\ \frac{3}{2} & -1 & \frac{1}{2} \end{pmatrix}$$

```
MatrixForm[m = Dot[%2, b]]
```

$$\begin{pmatrix} \frac{7}{6} \\ -\frac{1}{3} \\ \frac{1}{2} \end{pmatrix}$$

```
LinearSolve[{{1, 2, 1}, {2, 7, 4}, {1, 8, 7}}, b]
```

$$\left\{\frac{7}{6}, -\frac{1}{3}, \frac{1}{2}\right\}$$

```
Dot[{{1, 2, 1}, {2, 7, 4}, {1, 8, 7}}, %4]
```

```
{1, 2, 2}
```

```
LUDecomposition[{{1, 2, 1}, {2, 7, 4}, {1, 8, 7}}]
```

```
{{{1, 2, 1}, {2, 3, 2}, {1, 2, 2}}, {1, 2, 3}, 1}
```

```
Dot[{{1, 0, 0}, {2, 1, 0}, {1, 2, 1}}, {{1, 2, 1}, {0, 3, 2}, {0, 0, 2}}]
```

```
{{1, 2, 1}, {2, 7, 4}, {1, 8, 7}}
```

```
MatrixForm[Dot[Inverse[{{1, 2, 1}, {0, 3, 2}, {0, 0, 2}}],
  Inverse[{{1, 0, 0}, {2, 1, 0}, {1, 2, 1}}]]]
```

$$\begin{pmatrix} \frac{17}{6} & -1 & \frac{1}{6} \\ -\frac{5}{3} & 1 & -\frac{1}{3} \\ \frac{3}{2} & -1 & \frac{1}{2} \end{pmatrix}$$

example of using Det for cross product of vectors

```
Det[{{i, j, k}, {2, 3, 2}, {3, 2, 2}}]
```

$2\,i + 2\,j - 5\,k$

```
Cross[{2, 3, 2}, {3, 2, 2}]
```

```
{2, 2, -5}
```

Example of find 3x3 matrix eigensystem

```
In[1]:= m33 = {{1, 1, 3}, {1, 1, -3}, {3, -3, -3}};
```

solve eigen value use determinant, and solve eigen vectors use Solve (assume first component=1.)

```
In[2]:= Solve[Det[m33 - {{la, 0, 0}, {0, la, 0}, {0, 0, la}}] == 0, la]

Out[2]= {{la → -6}, {la → 2}, {la → 3}}

In[3]:= Solve[Dot[m33, {1, a, b}] == 3 {1, a, b}, {a, b}]

Out[3]= {{a → -1, b → 1}}

In[4]:= Solve[Dot[m33, {1, a, b}] == 2 {1, a, b}, {a, b}]

Out[4]= {{a → 1, b → 0}}

In[5]:= Solve[Dot[m33, {1, a, b}] == -6 {1, a, b}, {a, b}]

Out[5]= {{a → -1, b → -2}}

In[6]:= vec1 = {1, -1, -2};
        vec2 = {1, 1, 0};
        vec3 = {1, -1, 1};
```

test to see eigen system is correctly solved and normalize eigen vectors, unitary matrix with eigenvector as columns

```
In[7]:= Dot[m33, vec1] - (-6) * vec1
        Dot[m33, vec2] - 2 * vec2
        Dot[m33, vec3] - 3 * vec3

Out[7]= {0, 0, 0}

Out[8]= {0, 0, 0}

Out[9]= {0, 0, 0}

In[10]:= vec1 = vec1 / Sqrt[Dot[vec1, vec1]]
         vec2 = vec2 / Sqrt[Dot[vec2, vec2]]
         vec3 = vec3 / Sqrt[Dot[vec3, vec3]]
```

Out[10]= $\left\{\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\sqrt{\frac{2}{3}}\right\}$

Out[11]= $\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\}$

Out[12]= $\left\{\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}$

```
In[40]:= MatrixForm[mu = Transpose[{vec1, vec2, vec3}]]
         mut = Transpose[mu];
```

Out[40]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} \\ -\sqrt{\frac{2}{3}} & 0 & \frac{1}{\sqrt{3}} \end{pmatrix}$$

```
In[42]:= MatrixForm[Dot[mut, Dot[m33, mu]]]
```

Out[42]//MatrixForm=

$$\begin{pmatrix} -6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

solve eigen system use Eigensystem

```
In[13]:= Eigensystem[m33]

Out[13]= {{-6, 2, 3}, {{-1, 1, 2}, {1, 1, 0}, {1, -1, 1}}}

In[14]:= m = Last[%13];
```

Form unitary matrix

```
In[24]:= mod1 = Sqrt[Dot[Dot[{1, 0, 0}, m], Dot[{1, 0, 0}, m]]];
         mod2 = Sqrt[Dot[Dot[{0, 1, 0}, m], Dot[{0, 1, 0}, m]]];
         mod3 = Sqrt[Dot[Dot[{0, 0, 1}, m], Dot[{0, 0, 1}, m]]];

In[25]:= MatrixForm[mu = Dot[{{1 / mod1, 0, 0}, {0, 1 / mod2, 0}, {0, 0, 1 / mod3}}, m]]
```

Out[25]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

```
In[26]:= mut = Transpose[mu]
```

Out[26]= $\left\{\left\{-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}\right\}, \left\{\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{3}}\right\}, \left\{\sqrt{\frac{2}{3}}, 0, \frac{1}{\sqrt{3}}\right\}\right\}$

```
In[27]:= Dot[Dot[mu, m33], mut]

Out[27]= {{-6, 0, 0}, {0, 2, 0}, {0, 0, 3}}
```