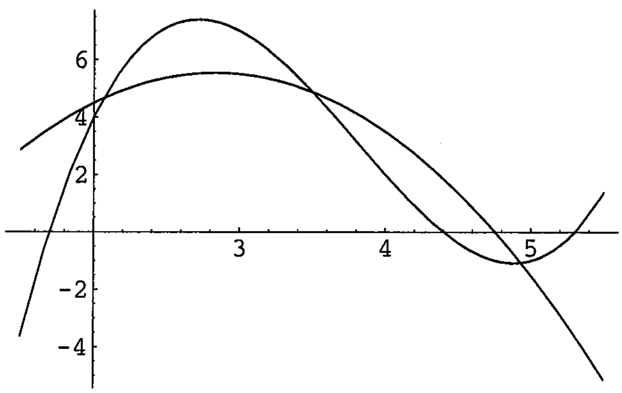
Example of fitting 5 points using Lagrange method and compare with polynomial fit to a lower order

```
pts = {{2, 4}, {3, 7}, {4, 2}, {5, -1}};

eqn = (x - 3) (x - 4) (x - 5) / ((2 - 3) (2 - 4) (2 - 5)) * 4 + (x - 2) (x - 4)
    (x - 5) / ((3 - 2) (3 - 4) (3 - 5)) * 7 + (x - 2) (x - 3) (x - 5) / ((4 - 2) (4 - 3) (4 - 5)) * 2 +
   (x - 2) (x - 3) (x - 4) / ((5 - 2) (5 - 3) (5 - 4)) * (-1);

lagran = Expand[eqn]
```

$$-66 + \frac{199\,x}{3} - 19\,x^2 + \frac{5\,x^3}{3}$$

```
polyfit = Fit[pts, {1, x, x^2}, x];

Plot[{polyfit, lagran}, {x, 1.5, 5.5}]
```



- Graphics -

Example of fit polynomial to a set of data

```
pts1 = {{-1, 3}, {0, 1}, {1, 4}};
polyfit = Fit[pts1, {1, z, z^2}, z];
```

Example of cubic spline through points (-1,3), (0,1), (1,4).

```
pts1 = {{-1, 3}, {0, 1}, {1, 4}};
f[x_] := a0 + a1 * x + a2 * x^2 + a3 * x^3
fp = D[f[x], x];
fpp = D[fp, x];

g[x_] := b0 + b1 * x + b2 * x^2 + b3 * x^3;
gp = D[g[x], x];
gpp = D[gp, x];

x = -1;
f1 = f[x];
fp1 = fp;
fpp1 = fpp;

General::spell1 :
 Possible spelling error: new symbol name "fpp1" is similar to existing symbol "fp1".
```

```
x = 0;
f2 = f[x];
fp2 = fp;
fpp2 = fpp;
g1 = g[x];
gp1 = gp;
gpp1 = gpp;
```

```
General::spell1 :
 Possible spelling error: new symbol name "fpp2" is similar to existing symbol "fp2".

General::spell :
 Possible spelling error: new symbol name "gpp1" is similar to existing symbols {fpp1, gp1}.
```

```
x = 1;
g2 = g[x];
gp2 = gp;
gpp2 = gpp;
```

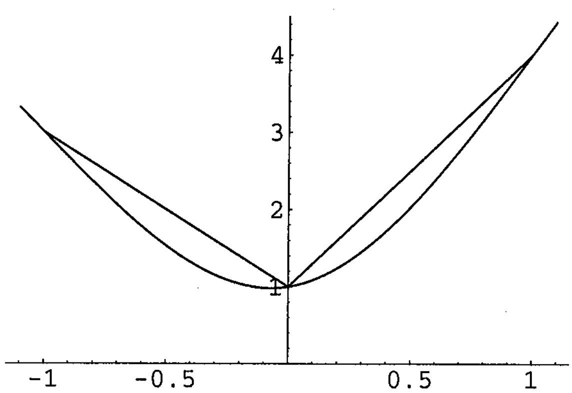
```
General::spell :
 Possible spelling error: new symbol name "gpp2" is similar to existing symbols {fpp2, gp2}.
```

```
sol = Solve[{f1 == 3, fpp1 == 0, f2 == 1, g1 == 1, fp2 == gp1, fpp2 == gpp1, g2 == 4, gpp2 == 0},
  {a0, a1, a2, a3, b0, b1, b2, b3}]
```

$$\left\{\left\{a0 \to 1,\ a1 \to \frac{1}{2},\ a2 \to \frac{15}{4},\ a3 \to \frac{5}{4},\ b0 \to 1,\ b1 \to \frac{1}{2},\ b2 \to \frac{15}{4},\ b3 \to -\frac{5}{4}\right\}\right\}$$

```
f[x_] := Last[a0 /. sol] + Last[a1 /. sol] * x + Last[a2 /. sol] * x^2 + Last[a3 /. sol] * x^3
g[x_] := Last[b0 /. sol] + Last[b1 /. sol] * x + Last[b2 /. sol] * x^2 + Last[b3 /. sol] * x^3
```

```
list1 = Table[{x, f[x]}, {x, 0, -1.1, -0.025}];
list2 = Table[{x, g[x]}, {x, 1.1, 0, -0.025}];
listail = Table[{x, g[x]}, {x, 1.0, 1.1, 0.025}];
list = Join[pts1, listail, list2, list1];
ListPlot[list, PlotRange -> {0, 4.5}, PlotJoined -> True]
```



- Graphics -

Rewrite a polynomial fraction into a polynomial plus a proper fraction. Further rewrite the proper fraction as sum of fraction of lowest possible order of polynomial fractions.
So $\frac{x^3+x-1}{x^2-3\,x+2}=3+x+\frac{-7+8\,x}{x^2-3\,x+2}=3+x+\frac{9}{-2+x}-\frac{1}{-1+x}$

```
In[1]:= PolynomialQuotient[x^3 + x - 1, x^2 - 3 x + 2, x]

Out[1]= 3 + x

In[2]:= PolynomialRemainder[x^3 + x - 1, x^2 - 3 x + 2, x]

Out[2]= -7 + 8 x

In[3]:= Factor[x^2 - 3 x + 2]

Out[3]= (-2 + x) (-1 + x)

In[4]:= SolveAlways[(x^3 + x - 1) / (x^2 - 3 x + 2) - 3 - x == A / (x - 2) + B / (x - 1), x]

Out[4]= {{A → 9, B → -1}}
```