

example of prove product of cyclic partial derivatives equals -1

Solve[**P V == R T Exp**[-**a1** / (**T V R**)], **P**]

Solve[**P V == R T Exp**[-**a1** / (**T V R**)], **V**]

Solve[**P V == R T Exp**[-**a1** / (**T V R**)], **T**]

$$\left\{ \left\{ P \rightarrow \frac{E^{-\frac{a1}{RTV}} R T}{V} \right\} \right\}$$

$$\left\{ \left\{ V \rightarrow -\frac{a1}{R T \text{ProductLog}\left[-\frac{a1 P}{R^2 T^2}\right]} \right\} \right\}$$

$$\left\{ \left\{ T \rightarrow \frac{a1}{R V \text{ProductLog}\left[\frac{a1}{P V^2}\right]} \right\} \right\}$$

$$\text{pro} = D\left[\frac{E^{-\frac{a1}{RTV}} R T}{V}, V\right] D\left[-\frac{a1}{R T \text{ProductLog}\left[-\frac{a1 P}{R^2 T^2}\right]}, T\right] D\left[\frac{a1}{R V \text{ProductLog}\left[\frac{a1}{P V^2}\right]}, P\right]$$

$$\left(a1 \left(\frac{a1 E^{-\frac{a1}{RTV}}}{V^3} - \frac{E^{-\frac{a1}{RTV}} R T}{V^2} \right) \right. \\ \left. \left(\frac{a1}{R T^2 \text{ProductLog}\left[-\frac{a1 P}{R^2 T^2}\right]} - \frac{2 a1}{R T^2 \text{ProductLog}\left[-\frac{a1 P}{R^2 T^2}\right] (1 + \text{ProductLog}\left[-\frac{a1 P}{R^2 T^2}\right])} \right) \right) / \\ \left(P R V \text{ProductLog}\left[\frac{a1}{P V^2}\right] (1 + \text{ProductLog}\left[\frac{a1}{P V^2}\right]) \right)$$

fun = **pro** /. **Exp**[-**a1** / (**R V T**)] -> **P V** / (**R T**) /. **ProductLog**[-**a1 P** / (**R^2 T^2**)] -> -**a1** / (**R T V**) /.
ProductLog[**a1** / (**P V^2**)] -> **a1** / (**R T V**)

$$\frac{T \left(\frac{a1 P}{R T V^2} - \frac{P}{V} \right) \left(-\frac{V}{T} + \frac{2 V}{T \left(1 - \frac{a1}{R T V} \right)} \right)}{P \left(1 + \frac{a1}{R T V} \right)}$$

Simplify[**fun**]

-1

Use example of reduced van der Waals gas equation prove $dv/dt = -dp/dt / dp/dv$ with numerical results comparison of the two expression. Works for parameters above critical value ($t, v, p > 1$) and some below critical value regions.

```
eqn := (p + 3 / v^2) (v - 1 / 3) == 8 / 3 * t;
sol = Solve[eqn, p]
```

```
{ {p -> - (3 - 9 v - 8 t v^2) / (v^2 (-1 + 3 v)) } }
```

```
solp = Last[p /. sol]
```

```
General::spell1 :
```

```
Possible spelling error: new symbol name "solp" is similar to existing symbol "sol".
```

```
- (3 - 9 v - 8 t v^2) / (v^2 (-1 + 3 v))
```

```
pt = D[solp, t]
```

```
8 / (-1 + 3 v)
```

```
pv = D[solp, v]
```

```
- (9 - 16 t v) / (v^2 (-1 + 3 v)) + (3 (-3 + 9 v - 8 t v^2)) / (v^2 (-1 + 3 v)^2) + (2 (-3 + 9 v - 8 t v^2)) / (v^3 (-1 + 3 v))
```

```
ptv = Simplify[-pt / pv]
```

```
(4 v^3 (-1 + 3 v)) / (3 (-1 + 6 v - 9 v^2 + 4 t v^3))
```

sol = Solve[eqn, v]

$$\left\{ \left\{ v \rightarrow -\frac{-p-8t}{9p} - (81p - (-p-8t)^2) / \left(9p \left(243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3} \right)^{(1/3)} \right) + \frac{1}{9p} \left((243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{(1/3)} \right) \right\}, \right. \\ \left. \left\{ v \rightarrow -\frac{-p-8t}{9p} + \left((1 + I\sqrt{3}) (81p - (-p-8t)^2) \right) / \left(18p \left(243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3} \right)^{(1/3)} \right) - \frac{1}{18p} \left((1 - I\sqrt{3}) (243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{(1/3)} \right) \right\}, \right. \\ \left. \left\{ v \rightarrow -\frac{-p-8t}{9p} + \left((1 - I\sqrt{3}) (81p - (-p-8t)^2) \right) / \left(18p \left(243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3} \right)^{(1/3)} \right) - \frac{1}{18p} \left((1 + I\sqrt{3}) (243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{(1/3)} \right) \right\} \right\}$$

solv = First[v /. sol]

General::spell :

Possible spelling error: new symbol name "solv" is similar to existing symbols {sol, solp}.

$$-\frac{-p-8t}{9p} - (81p - (-p-8t)^2) / \left(9p \left(243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3} \right)^{(1/3)} \right) + \frac{1}{9p} \left((243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{(1/3)} \right)$$

pptt = D[solv, t]

$$\begin{aligned} & \frac{8}{9p} + \left((81p - (-p - 8t)^2) \left(-972p + 24p^2 + 384pt + \right. \right. \\ & \quad \left. \left. 1536t^2 + \frac{27p(-1080p + 24p^2 - 864t + 384pt + 1536t^2)}{2\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3}} \right) \right) / \\ & \left(27p(243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + \right. \\ & \quad \left. 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{(4/3)} \right) + \\ & \left(-972p + 24p^2 + 384pt + \right. \\ & \quad \left. 1536t^2 + \frac{27p(-1080p + 24p^2 - 864t + 384pt + 1536t^2)}{2\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3}} \right) / \\ & \left(27p(243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + \right. \\ & \quad \left. 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{(2/3)} \right) - \\ & (16(-p - 8t)) / (9p(243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + \\ & \quad 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{(1/3)}) \end{aligned}$$

ptvv = ptv /. v -> solv

General::spell1 :

Possible spelling error: new symbol name "ptvv" is similar to existing symbol "ptv".

$$\left(4 \left(-\frac{-p-8t}{9p} - (81p - (-p-8t)^2) / (9p (243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{1/3}\right) + \frac{1}{9p} \left((243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{1/3}\right)\right)^3 \right. \\ \left. \left(-1 + 3 \left(-\frac{-p-8t}{9p} - (81p - (-p-8t)^2) / (9p (243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{1/3}\right) + \frac{1}{9p} \left((243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{1/3}\right)\right)\right) / \right. \\ \left. \left(3 \left(-1 + 6 \left(-\frac{-p-8t}{9p} - (81p - (-p-8t)^2) / (9p (243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{1/3}\right) + \frac{1}{9p} \left((243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{1/3}\right)\right) - \right. \right. \\ \left. \left. 9 \left(-\frac{-p-8t}{9p} - (81p - (-p-8t)^2) / (9p (243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{1/3}\right) + \frac{1}{9p} \left((243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{1/3}\right)\right) \right)^2 + \right. \\ \left. 4t \left(-\frac{-p-8t}{9p} - (81p - (-p-8t)^2) / (9p (243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{1/3}\right) + \frac{1}{9p} \left((243p^2 + p^3 - 972pt + 24p^2t + 192pt^2 + 512t^3 + 27p\sqrt{729p + 54p^2 + p^3 - 1080pt + 24p^2t - 432t^2 + 192pt^2 + 512t^3})^{1/3}\right)\right) \right)^3 \right)$$

t = 1.1;

v = 0.75;

solv = Solve[eqn, p]

{{p -> 1.70667}}

p = Last[p /. sol]

1.70667

pptt
ptv
ptvv

2.39362

2.39362

2.39362