

(* question 1 *)

Solve[{Dt[Sin[y] - Exp[x], x] == 0, Sin[y] == Exp[x]}, Dt[y, x], y]

Solve::bdomv : Warning: y is not a valid domain specification. Assuming it is a variable to eliminate. >>

Solve::ifun :

Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$$\text{Out[32]= } \left\{ \left\{ \text{Dt}[y, x] \rightarrow -\frac{i e^x}{\sqrt{-1 + e^{2x}}} \right\}, \left\{ \text{Dt}[y, x] \rightarrow \frac{i e^x}{\sqrt{-1 + e^{2x}}} \right\} \right\}$$

(* question 2 *)

FindRoot[x² == Tanh[x], {x, 1.1}]

$$\text{Out[2]= } \{x \rightarrow 0.822429\}$$

In[3]:= (* question 3 *)

x = a Cos[t];

y = b Sin[t];

Tan[θ] = $\frac{y}{x}$;

Set::write : Tag Tan in Tan[θ] is Protected. >>

In[8]:= **Solve**[{0 == Dt[Tan[θ] - $\frac{y}{x}$, t] /. Dt[a, t] → 0 /. Dt[b, t] → 0, Tan[θ] == $\frac{y}{x}$ }, Dt[θ, t], θ]

Solve::bdomv : Warning: θ is not a valid domain specification. Assuming it is a variable to eliminate. >>

Solve::ifun :

Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$$\text{Out[8]= } \left\{ \left\{ \text{Dt}[\theta, t] \rightarrow \frac{a b \text{Sec}[t]^2}{a^2 + b^2 \text{Tan}[t]^2} \right\} \right\}$$

In[9]:= **area** = 4 Integrate[$\frac{1}{2} (x^2 + y^2) \text{Dt}[\theta, t] /. \text{Dt}[\theta, t] \rightarrow \frac{a b \text{Sec}[t]^2}{a^2 + b^2 \text{Tan}[t]^2}$, {t, 0, $\frac{\pi}{2}}$]

$$\text{Out[9]= } a b \pi$$

In[10]:= (* question 5 *)

Solve[{m₁ v₁ + m₂ v₂ == m₁ v_A + m₂ v_B, $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 == \frac{1}{2} m_1 v_A^2 + \frac{1}{2} m_2 v_B^2$ }, {v_A, v_B}]

$$\text{Out[10]= } \left\{ \left\{ v_A \rightarrow v_1, v_B \rightarrow v_2 \right\}, \left\{ v_A \rightarrow \frac{m_1 v_1 - m_2 v_1 + 2 m_2 v_2}{m_1 + m_2}, v_B \rightarrow \frac{2 m_1 v_1 - m_1 v_2 + m_2 v_2}{m_1 + m_2} \right\} \right\}$$

In[13]:= **Simplify**[(v_B - v_A) /. v_A → $\frac{m_1 v_1 - m_2 v_1 + 2 m_2 v_2}{m_1 + m_2}$ /. v_B → $\frac{2 m_1 v_1 - m_1 v_2 + m_2 v_2}{m_1 + m_2}$]

$$\text{Out[13]= } v_1 - v_2$$

In[15]:= (* question 6 *)

Solve[{ $m_1 g - T_1 == m_1 a_1$, $m_2 g - T_2 == m_2 a_2$, $m_3 g - T_2 == m_3 a_3$, $T_1 == 2 T_2$, $2 a_1 + a_2 + a_3 == 0$ },
 $\{a_1, a_2, a_3, T_1\}$, $\{T_2\}$]

Out[15]= $\left\{ \left\{ a_1 \rightarrow -\frac{-g m_1 m_2 - g m_1 m_3 + 4 g m_2 m_3}{m_1 m_2 + m_1 m_3 + 4 m_2 m_3}, a_2 \rightarrow -\frac{-g m_1 m_2 + 3 g m_1 m_3 - 4 g m_2 m_3}{m_1 m_2 + m_1 m_3 + 4 m_2 m_3}, \right. \right.$
 $\left. a_3 \rightarrow -\frac{3 g m_1 m_2 - g m_1 m_3 - 4 g m_2 m_3}{m_1 m_2 + m_1 m_3 + 4 m_2 m_3}, T_1 \rightarrow \frac{8 g m_1 m_2 m_3}{m_1 m_2 + m_1 m_3 + 4 m_2 m_3} \right\}$

In[16]:= **Simplify**[$(a_1 m_1 + a_2 m_2 + a_3 m_3 + 2 T_1) /. a_1 \rightarrow -\frac{-g m_1 m_2 - g m_1 m_3 + 4 g m_2 m_3}{m_1 m_2 + m_1 m_3 + 4 m_2 m_3} /.$
 $a_2 \rightarrow -\frac{-g m_1 m_2 + 3 g m_1 m_3 - 4 g m_2 m_3}{m_1 m_2 + m_1 m_3 + 4 m_2 m_3} /.$
 $a_3 \rightarrow -\frac{3 g m_1 m_2 - g m_1 m_3 - 4 g m_2 m_3}{m_1 m_2 + m_1 m_3 + 4 m_2 m_3} /. T_1 \rightarrow \frac{8 g m_1 m_2 m_3}{m_1 m_2 + m_1 m_3 + 4 m_2 m_3}]$

Out[16]= $g (m_1 + m_2 + m_3)$