In[1]:=
```
(* lagrange polynomial *)
data = {{-1, 3}, {0, 1}, {1, 4}};
```

In[2]:=
```
Fit[data, {1, x, x^2}, x]
```

Out[2]=
$$1. + 0.5\,x + 2.5\,x^2$$

In[3]:=
```
sol = (x - 0) (x - 1) / ((-1 - 0) (-1 - 1)) 3 + (x + 1) (x - 1) / ((0 + 1) (0 - 1)) 1 + (x + 1) (x - 0) / ((1 + 1) (1 - 0)) 4;
Expand[sol]
```

Out[4]=
$$1 + \frac{x}{2} + \frac{5\,x^2}{2}$$

In[5]:=
```
(* polynomial fit vs. cubic spline. data must be in {t,x} form *)
data = {{0, -3}, {1, -1}, {2, 0}, {3, 2}, {4, 1}, {5, 7}};
```

In[6]:=
```
pol = Fit[data, {1, t, t^2, t^3, t^4, t^5}, t]
```

Out[6]=
$$-3. + 8.66667\,t - 12.5833\,t^2 + 7.66667\,t^3 - 1.91667\,t^4 + 0.166667\,t^5$$

In[7]:=
```
f1[t] = a0 + a1 t + a2 t^2 + a3 t^3;
f2[t] = b0 + b1 t + b2 t^2 + b3 t^3;
f3[t] = c0 + c1 t + c2 t^2 + c3 t^3;
f4[t] = d0 + d1 t + d2 t^2 + d3 t^3;
f5[t] = e0 + e1 t + e2 t^2 + e3 t^3;
```

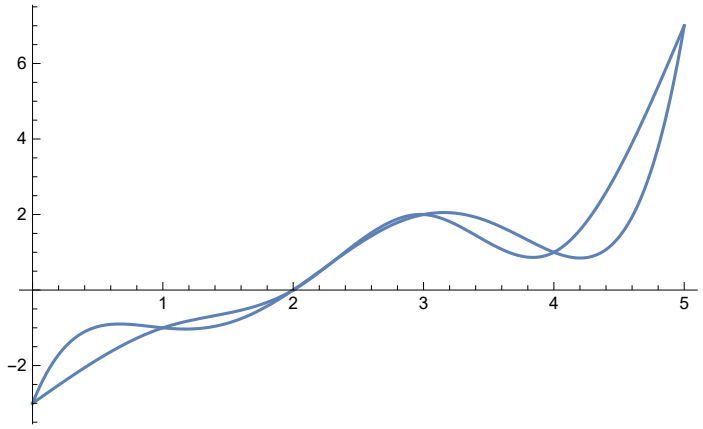
In[12]:=
```
Solve[{-3 == f1[t] /. t → 0, -1 == f1[t] /. t → 1, -1 == f2[t] /. t → 1, 0 == f2[t] /. t → 2,
  0 == f3[t] /. t → 2, 2 == f3[t] /. t → 3, 2 == f4[t] /. t → 3, 1 == f4[t] /. t → 4,
  1 == f5[t] /. t → 4, 7 == f5[t] /. t → 5, 0 == D[D[f1[t], t], t] /. t → 0,
  0 == D[D[f5[t], t], t] /. t → 5, (D[f1[t], t] /. t → 1) == (D[f2[t], t] /. t → 1),
  (D[D[f1[t], t], t] /. t → 1) == (D[D[f2[t], t], t] /. t → 1),
  (D[f2[t], t] /. t → 2) == (D[f3[t], t] /. t → 2),
  (D[D[f2[t], t], t] /. t → 2) == (D[D[f3[t], t], t] /. t → 2),
  (D[f3[t], t] /. t → 3) == (D[f4[t], t] /. t → 3),
  (D[D[f3[t], t], t] /. t → 3) == (D[D[f4[t], t], t] /. t → 3),
  (D[f4[t], t] /. t → 4) == (D[f5[t], t] /. t → 4),
  (D[D[f4[t], t], t] /. t → 4) == (D[D[f5[t], t], t] /. t → 4)
 }, {a0, a1, a2, a3, b0, b1, b2, b3, c0, c1, c2, c3, d0, d1, d2, d3, e0, e1, e2, e3}]
```

Out[12]=
$$\left\{\left\{a0 \to -3,\ a1 \to \frac{508}{209},\ a2 \to 0,\ a3 \to -\frac{90}{209},\ b0 \to -\frac{958}{209},\ b1 \to \frac{79}{11},\ b2 \to -\frac{993}{209},\ b3 \to \frac{241}{209},\right.\right.$$
$$c0 \to \frac{4618}{209},\ c1 \to -\frac{6863}{209},\ c2 \to \frac{3189}{209},\ c3 \to -\frac{24}{11},\ d0 \to -\frac{2533}{19},\ d1 \to \frac{25\,618}{209},$$
$$\left.\left.d2 \to -\frac{402}{11},\ d3 \to \frac{747}{209},\ e0 \to \frac{48\,233}{209},\ e1 \to -\frac{31\,454}{209},\ e2 \to \frac{6630}{209},\ e3 \to -\frac{442}{209}\right\}\right\}$$

In[13]:=
```
g1 = f1[t] /. a0 → -3 /. a1 → 508/209 /. a2 → 0 /. a3 → -90/209;
g2 = f2[t] /. b0 → -958/209 /. b1 → 79/11 /. b2 → -993/209 /. b3 → 241/209;
g3 = f3[t] /. c0 → 4618/209 /. c1 → -6863/209 /. c2 → 3189/209 /. c3 → -24/11;
g4 = f4[t] /. d0 → -2533/19 /. d1 → 25 618/209 /. d2 → -402/11 /. d3 → 747/209;
g5 = f5[t] /. e0 → 48 233/209 /. e1 → -31 454/209 /. e2 → 6630/209 /. e3 → -442/209;
```

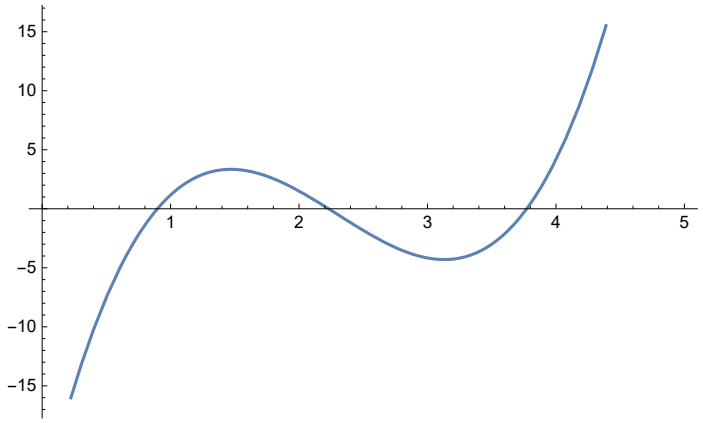
In[18]:=
```
Show[Plot[pol, {t, 0, 5}], Plot[g1, {t, 0, 1}], Plot[g2, {t, 1, 2}],
 Plot[g3, {t, 2, 3}], Plot[g4, {t, 3, 4}], Plot[g5, {t, 4, 5}]]
```

Out[18]=



In[19]:=
```
polf = D[D[pol, t], t];
Plot[polf, {t, 0, 5}]
```

Out[20]=



In[21]:=
```
g1f = D[D[g1, t], t];
g2f = D[D[g2, t], t];
g3f = D[D[g3, t], t];
g4f = D[D[g4, t], t];
g5f = D[D[g5, t], t];
Show[Plot[polf, {t, 0, 5}], Plot[g1f, {t, 0, 1}], Plot[g2f, {t, 1, 2}],
 Plot[g3f, {t, 2, 3}], Plot[g4f, {t, 3, 4}], Plot[g5f, {t, 4, 5}], PlotRange → All]
```

Out[26]=