HW-3 #1

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In[1]:= x = L Sin[θ[t]];
        y = 1/2 c t^2 - L Cos[θ[t]];

In[9]:= lag = FullSimplify[m/2 (D[x, t]^2 + D[y, t]^2) - m g y]
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Out[9]= $\frac{1}{2} m \left(c (c-g) t^2 + 2 g L \mathrm{Cos}[\theta[t]] + L \theta'[t] (2 c t \mathrm{Sin}[\theta[t]] + L \theta'[t])\right)$

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In[4]:= Solve[Pθ[t] == D[lag, θ'[t]], θ'[t]]
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Out[4]= $\left\{\left\{\theta'[t] \to \frac{-c L m t \mathrm{Sin}[\theta[t]] + P_\theta[t]}{L^2 m \left(\mathrm{Cos}[\theta[t]]^2 + \mathrm{Sin}[\theta[t]]^2\right)}\right\}\right\}$

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In[6]:= hamil =
         FullSimplify[(D[lag, θ'[t]] θ'[t] - lag) /. θ'[t] → (-c L m t Sin[θ[t]] + Pθ[t])/(L^2 m (Cos[θ[t]]^2 + Sin[θ[t]]^2))]
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Out[6]= $\frac{1}{4 L^2 m}\left(-L^2 m^2 \left(4 g L \mathrm{Cos}[\theta[t]] + c t^2 (c - 2 g + c \mathrm{Cos}[2 \theta[t]])\right) + 2 P_\theta[t] (-2 c L m t \mathrm{Sin}[\theta[t]] + P_\theta[t])\right)$

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In[7]:= (* legendre transformation one more time back *)
        Solve[θ'[t] == D[hamil, Pθ[t]], Pθ[t]]
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Out[7]= $\{\{P_\theta[t] \to L m (c t \mathrm{Sin}[\theta[t]] + L \theta'[t])\}\}$

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In[8]:= lagnew =
         FullSimplify[(D[hamil, Pθ[t]] Pθ[t] - hamil) /. Pθ[t] → L m (c t Sin[θ[t]] + L θ'[t])]
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Out[8]= $\frac{1}{2} m \left(c (c-g) t^2 + 2 g L \mathrm{Cos}[\theta[t]] + L \theta'[t] (2 c t \mathrm{Sin}[\theta[t]] + L \theta'[t])\right)$

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In[10]:= lag - lagnew
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Out[10]= 0

HW-3

2. $P = \frac{8}{3} T \frac{1}{V - \frac{1}{3}} - \frac{3}{V^2}$

$\beta = \frac{1}{V} \frac{\partial V}{\partial T}\Big)_P$, $\frac{\partial V}{\partial T}\Big)_P \cdot \frac{\partial P}{\partial V}\Big)_T \cdot \frac{\partial T}{\partial P}\Big)_V = -1$

$$\frac{\partial V}{\partial T}\Big)_P = - \frac{\frac{\partial P}{\partial T}\big)_V}{\frac{\partial P}{\partial V}\big)_T} = \frac{\frac{8}{3} \frac{1}{V - \frac{1}{3}}}{\frac{8}{3} T \frac{-1}{(V - \frac{1}{3})^2} + \frac{6}{V^3}}$$

so: $\beta = \frac{4V^3(3V-1)}{3(4TV^3 - 9V^2 + 6V - 1)}$

4. if $\frac{\partial A}{\partial y}\Big)_x = \frac{\partial B}{\partial x}\Big)_y$

and $\frac{\partial x}{\partial y}\Big)_A \frac{\partial A}{\partial x}\Big)_y \frac{\partial y}{\partial A}\Big)_x = -1$

$$\frac{\partial x}{\partial y}\Big)_A = - \frac{\frac{\partial A}{\partial y}\big)_x}{\frac{\partial A}{\partial x}\big)_y} = - \frac{\frac{\partial B}{\partial x}\big)_y}{\frac{\partial A}{\partial x}\big)_y} = - \frac{\partial B}{\partial A}\Big)_y$$

5. $H = U + PV$ $\quad C_p = \frac{\partial H}{\partial T}\Big)_p = \frac{\partial U}{\partial T}\Big)_p + P \frac{\partial V}{\partial T}\Big)_p$

and $\frac{\partial U}{\partial T}\Big)_p = \frac{\partial U}{\partial T}\Big)_V + \frac{\partial U}{\partial V}\Big)_T \frac{\partial V}{\partial T}\Big)_p = C_V + \frac{\partial U}{\partial V}\Big)_T \frac{\partial V}{\partial T}\Big)_p$

so: $C_p - C_V = \frac{\partial V}{\partial T}\Big)_p \left(\frac{\partial U}{\partial V}\Big)_T + P \right)$

use energy equation $\quad C_p - C_V = T \frac{\partial P}{\partial T}\Big)_V \frac{\partial V}{\partial T}\Big)_p$

4a) $dz = A\,dx + B\,dy + x\,dA - x\,dA$

$d(z - xA) = B\,dy - x\,dA$

therefore $\frac{\partial B}{\partial A}\Big)_y = - \frac{\partial x}{\partial y}\Big)_A$

#6. $f(x,y)$ and $u = x^2 - y^2$, $v = 2xy$

$$\left.\frac{\partial f}{\partial x}\right)_y = \left.\frac{\partial f}{\partial v}\right)_u \left.\frac{\partial v}{\partial x}\right)_y + \left.\frac{\partial f}{\partial u}\right)_v \left.\frac{\partial u}{\partial x}\right)_y$$

or $$\left.\frac{\partial f}{\partial x}\right)_y = 2y \left.\frac{\partial f}{\partial v}\right)_u + 2x \left.\frac{\partial f}{\partial u}\right)_v$$

$$\left.\frac{\partial f}{\partial y}\right)_x = \left.\frac{\partial f}{\partial v}\right)_u \left.\frac{\partial v}{\partial y}\right)_x + \left.\frac{\partial f}{\partial u}\right)_v \left.\frac{\partial u}{\partial y}\right)_x$$

or $$\left.\frac{\partial f}{\partial y}\right)_x = 2x \left.\frac{\partial f}{\partial v}\right)_u - 2y \left.\frac{\partial f}{\partial u}\right)_v$$

since $$y \left.\frac{\partial f}{\partial x}\right)_y + x \left.\frac{\partial f}{\partial y}\right)_x = 0$$

$$0 = 2(x^2+y^2) \left.\frac{\partial f}{\partial v}\right)_u = 0$$

therefore $$\left.\frac{\partial f}{\partial v}\right)_u = 0 \ !$$