HW-4

1) Carbon $2P^2$ 2 eletron in 6-states

$$\frac{6!}{2!4!} = 15 \text{ states}$$

2 spin parallel 3 states for 2 ele.

$$\frac{3!}{2!1!} = 3 \text{ states } |\uparrow\uparrow\rangle$$

3 states $|\downarrow\downarrow\rangle$

3 states $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$

9 states parallel spin (symmetric spin wavefunctions)

$15 - 9 = 6$ states spin anti parallel

spin singlet $2J+1 = 1, 3, 5 \ldots$

$6 = 1+5$ $\quad {}^1S_0 \quad {}^1D_2$

$9 = 1+3+5$ $\quad {}^3P_0 \quad {}^3P_1 \quad {}^3P_2$

2)
$$\frac{\Omega_{BE}}{\Omega_{FD}} = \frac{(N+n-1)!}{n!(N-1)!} \frac{n!(N-n)!}{N!} = \frac{N}{N+n} \frac{(N+n)!}{N!} \cdot \frac{(N-n)!}{N!}$$

$$= \frac{N}{N+n} \frac{(N+1)(N+2)\cdots(N+n)}{(N+1-n)(N+2-n)\cdots N} = \prod_{i=1}^{n-1} \frac{N+i}{N+i-n} > 1 \quad (n \geq 1)$$

when $N \gg n$ $\quad \frac{(N+n-1)!}{(N-1)!} \gtrsim N^n, \quad \frac{N!}{(N-n)!} \lesssim N^n$

so $\Omega_{BE} \sim \Omega_{FD} \sim \Omega_{MB} = \frac{N^n}{n!}$

$\boxed{HW\ 4}$ (EM 16)

16-3 $\left.\begin{matrix} 1,2,3,4 \\ 1,2,3,4 \end{matrix}\right\}$ 16 states

$Pro(2) = \frac{1}{16},\ Pro(3) = \frac{1}{8},\ Pro(4) = \frac{3}{16},\ Pro(5) = \frac{1}{4}$

$Pro(6) = \frac{3}{16},\ Pro(7) = \frac{1}{8},\ Pro(8) = \frac{1}{16}$

(a) $\langle win \rangle = \sum_{i=2}^{8} \frac{1}{7} i^2 P^A_{ro}(i) = 3.93$

$\langle loss \rangle = \sum_{i=2}^{8} \frac{1}{7} i\,(1 - P^A_{ro}(i)) = 4.29$ > -0.36

b) $\langle win \rangle = \sum P^B_{ro}(i)\, i^2 P^A_{ro}(i) = 4.56$

$\langle loss \rangle = \sum P^B_{ro}(i)\, i\,(1 - P^A_{ro}(i)) = 4.14$ > 0.42

c) $\max Pro(i) = Pro(5) = \frac{1}{4}$ $\quad x = 5$

$\langle win \rangle = 25\, Pro(5) = 6.25$

$\langle loss \rangle = 5\,(1 - Pro(5)) = 3.75$ > 2.50

But one can increase $\langle win \rangle$

$6^2\, Pro(6) = 6.75$

$\langle loss \rangle = 6\,(1 - Pro(6)) = 4.875$ > 1.875 !

HW 4

16-3 $\left.\begin{matrix}1,2,3,4\\1,2,3,4\end{matrix}\right\}$ 16 states

$Pro(2)=\frac{1}{16}$, $Pro(3)=\frac{1}{8}$; $Pro(4)=\frac{3}{16}$, $Pro(5)=\frac{1}{4}$;

$Pro(6)=\frac{3}{16}$; $Pro(7)=\frac{1}{8}$; $Pro(8)=\frac{1}{16}$

a) $\langle win\rangle = \sum_{i=2}^{8}\frac{1}{7} * Pro(i) * i^2 = 3.93$

$\langle loss\rangle = \sum_{i=2}^{8}\frac{1}{7}(1-Pro(i))\, i = 4.29$

b) $\langle win\rangle = \sum_{i=2}^{8} Pro^A(i)\, Pro^B(i)\, i^2 = 4.56$

$\langle loss\rangle = \sum_{i=2}^{8} Pro^A(i)\,(1-Pro^B(i))\, i = 4.14$

c) $\max[Pro(i)] = Pro(5) = \frac{1}{4}$ $X=5$

$\langle win\rangle = Pro(5) * 25 = 6.25$

$\langle loss\rangle = 5(1-Pro(5)) = 3.75$

☆, $\because \max[Pro(i)\, i^2 - (1-Pro(i))\, i]$ is when $i=5$

$\frac{3}{16} * 36 - (1-\frac{3}{16})6 = 1.875$

$\frac{1}{4} * 25 - (1-\frac{1}{4})5 = 2.50$

$\frac{3}{16} * 16 - (1-\frac{3}{16})4 = -0.25$ maybe

if win is $i^{2.5}$ then $\max[Pro(i)\, i^{2.5} - (1-Pro(i))\, i]$ @ $i=6$

having boy $= p$ girl $1-p = q$

$$Pro(l) = p(1-p)^{l-1} \qquad l = 1, 2, \ldots$$

$$\sum_{l=1}^{\infty} Pro(l) = p \sum_{l=0}^{\infty} (1-p)^l = p \frac{1}{1-(1-p)} = 1$$

$$\langle l \rangle = \sum_{l=1}^{\infty} l \, Pro(l) = p \sum_{l=1}^{\infty} l (1-p)^{l-1}$$

$$\langle l \rangle = p \frac{\partial}{\partial q} \sum_{l=1}^{\infty} q^l = p \frac{\partial}{\partial q} \left(q \sum_{l=0}^{\infty} q^l \right)$$

$$\langle l \rangle = p \frac{\partial}{\partial q} \left(\frac{q}{1-q} \right) = p \left(\frac{1}{1-q} + \frac{q}{(1-q)^2} \right)$$

$$\langle l \rangle = \frac{p}{(1-q)^2} = \frac{1}{p}$$

16-12 $\quad f(t) = 2t$

1 of n $\quad \frac{n!}{(n-1)!} 2t \int_0^t 2t_1\,dt_1 \int_0^t 2t_2\,dt_2 \cdots$

check: $\quad 2nt\,t^{2(n-1)} = 2n\,t^{2n-1} = f_n(t)$

$$\int_0^1 f_n(t)\,dt = 1$$

$$\langle t^2 \rangle = 2n \int_0^1 t^2 t^{2n-1}\,dt = \frac{2n}{2n+2} = \frac{n}{n+1}$$

$$\text{area} = \frac{n}{n+1}\pi$$

omit largest t, $(n-2)$ less than "2" $\quad \frac{n!}{(n-2)!} 2t\, t^{2(n-2)} \int_t^1 2t_2\,dt_2$

$$f(t) = 2n(n-1)\,t^{2n-3}(1-t^2)$$

check: $\int_0^1 f(t)\,dt = 2n(n-1)\left[\frac{1}{2n-2} - \frac{1}{2n}\right] = 1$

$$\text{area} = \pi\langle t^2\rangle = \pi\int_0^1 2n(n-1)\,t^{2n-1}(1-t^2)\,dt$$

$$= \pi\, 2n(n-1)\left[\frac{1}{2n} - \frac{1}{2n+2}\right] = \frac{n-1}{n+1}\pi$$

$$f(t) = 2 * \frac{n!}{(n-2)!\,2!}\, 2t \left[\int_0^t 2t'\,dt'\right]^{n-2} \int_t^1 2t'\,dt'$$

16 - 13

$$f(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{2} & 0 \le t \le 1 \\ k e^{-2t} & t \ge 1 \end{cases}$$

$$\int f(t)\,dt = 1 = \int_0^1 \tfrac{1}{2}\,dt + \int_1^\infty k e^{-2t}\,dt$$

$$1 = \tfrac{1}{2} + k \tfrac{1}{2} e^{-2t}\Big|_\infty^1 = \tfrac{1}{2} + \tfrac{1}{2} k e^{-2}; \quad \underline{\underline{k = e^2}}$$

$$\langle \$ \rangle = 20 * \int_0^{0.5} f(t)\,dt + 40 \int_{0.5}^{1} f(t)\,dt + 60 \int_{1.0}^{1.5} f(t)\,dt + \cdots$$

$$= \tfrac{1}{4} * 20 + \tfrac{1}{4} * 40 + 60 * \tfrac{1}{2} e^2 \left[e^{-2*(1.0)} - e^{-2*(1.5)} \right] + \cdots$$

$$= 15 + 30 e^2 e^{-2*1.0} (1 - e^{-1}) + 40 e^2 (1 - e^{-1}) e^{-2*2.0} + \cdots$$

$$= 15 + 20 e^2 (1 - e^{-1}) \sum_{i=1}^{\infty} e^{-2i} + 10 e^2 (1 - e^{-1}) \sum_{i=1}^{\infty} i e^{-2i}$$

$$\sum_{i=1}^{\infty} e^{-2i} = \sum_{i=1}^{\infty} q^i \qquad (q = e^{-2})$$

$$= \frac{1}{1-q} - 1 = \frac{q}{1-q}$$

$$\sum_{i=1}^{\infty} i e^{-2i} = \sum_{i=1}^{\infty} i q^i = \sum_{i=1}^{\infty} (i+1) q^i - \sum_{i=1}^{\infty} q^i$$

$$= \frac{\partial}{\partial q} \sum_{i=1}^{\infty} q^{i+1} - \frac{q}{1-q} = \frac{\partial}{\partial q} \sum_{i=0}^{\infty} q^i - \frac{q}{1-q}$$

$$= \frac{\partial}{\partial q} \frac{1}{1-q} - \frac{q}{1-q} = \frac{1}{(1-q)^2} - \frac{q}{1-q}$$

$$\langle \$ \rangle = 15 + 20 e^2 (1 - e^{-1}) \frac{e^{-2}}{1 - e^{-2}} + 10 e^2 (1 - e^{-1}) \left\{ \frac{1}{(1 - e^{-2})^2} - \frac{e^{-2}}{1 - e^{-2}} \right\}$$

$$\langle \$ \rangle = 15 + 10 e^2 (1 - e^{-1}) \left(\frac{1}{(1 - e^{-2})^2} + \frac{e^{-2}}{1 - e^{-2}} \right) = 84.8$$