(4-5) $\quad f(x) = x \qquad -\pi < x < \pi$

$$f(x) = \Sigma\, b_n \sin\frac{2n\pi}{2\pi}x = \Sigma\, b_n \sin nx$$

$$b_n = \frac{\int_{-\pi}^{\pi} x \sin nx\, dx}{\int_{-\pi}^{\pi} (\sin nx)^2 dx} = \frac{2}{\pi}\int_0^{\pi} x \sin(nx) dx$$

$$\int_0^{\pi} x \sin nx\, dx = -\frac{x}{n}\cos nx\Big|_0^{\pi} + \frac{1}{n}\int_0^{\pi} \cos nx\, dx$$

$$= -\frac{\pi}{n}(-1)^n + \cancel{\frac{1}{n^2}\sin nx\Big|_0^{\pi}}^{\,0}$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1}\sin nx$$

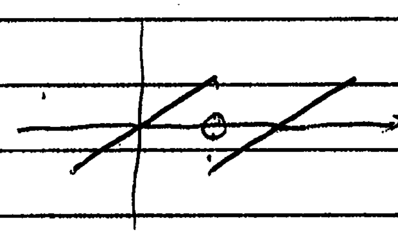
let $x = \frac{\pi}{2}$ and we get:

$$\frac{\pi}{4} = \Sigma\, \frac{(-1)^{n+1}}{n}\sin n\frac{\pi}{2} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

let $x = \pi$

$$\frac{\pi}{2} = \Sigma\, \frac{(-1)^{n+1}}{n}\sin n\pi$$

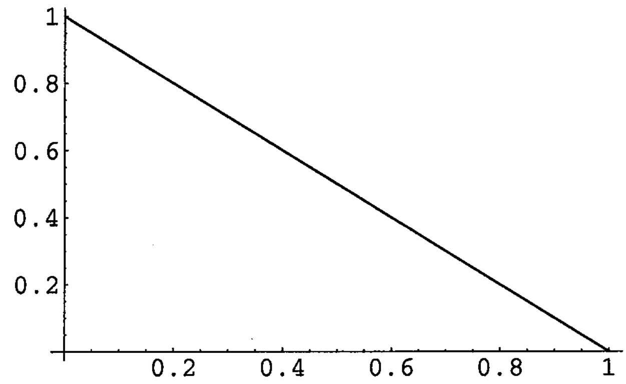
right side = zero! reason:



Fourier Series for f[x]=1-x, with even or odd functions up to 5 terms, and show even extension is better.

```
In[1]:= f[x_] := 1 - x;
        nmax = 5;
```

```
In[2]:= Plot[f[x], {x, 0, 1}]
```



```
Out[2]= - Graphics -
```

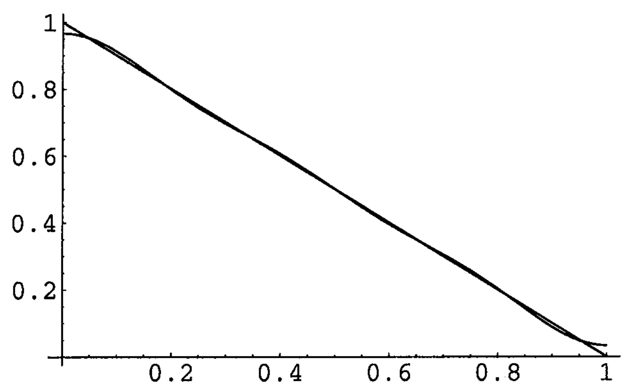
```
In[3]:= b0 = Integrate[f[x], {x, 0, 1}];
        bn = Table[Sqrt[2] * Integrate[Cos[n π x] f[x], {x, 0, 1}], {n, 1, nmax}];
        bfun = Table[Sqrt[2] * Cos[n π x], {n, 1, nmax}];
```

```
In[4]:= fun[x_] := b0 + Dot[bn, bfun]
```

```
In[5]:= fun[x]
```

Out[5]= $\frac{1}{2} + \frac{4\,\mathrm{Cos}[\pi x]}{\pi^2} + \frac{4\,\mathrm{Cos}[3\,\pi x]}{9\,\pi^2} + \frac{4\,\mathrm{Cos}[5\,\pi x]}{25\,\pi^2}$

```
In[6]:= Plot[{f[x], fun[x]}, {x, 0, 1}]
```

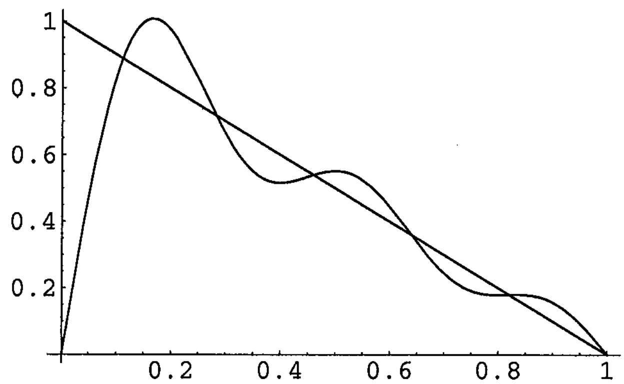


```
Out[6]= - Graphics -
```

```
In[7]:= an = Table[Sqrt[2] * Integrate[Sin[n π x] f[x], {x, 0, 1}], {n, 1, nmax}];
        afun = Table[Sqrt[2] * Sin[n π x], {n, 1, nmax}];

        General::spell :
         Possible spelling error: new symbol name "afun" is similar to existing symbols {bfun, fun}.

In[8]:= fun[x_] := Dot[an, afun]

In[9]:= fun[x]
```

Out[9]= $\frac{2\,\mathrm{Sin}[\pi\,x]}{\pi} + \frac{\mathrm{Sin}[2\,\pi\,x]}{\pi} + \frac{2\,\mathrm{Sin}[3\,\pi\,x]}{3\,\pi} + \frac{\mathrm{Sin}[4\,\pi\,x]}{2\,\pi} + \frac{2\,\mathrm{Sin}[5\,\pi\,x]}{5\,\pi}$

```
In[10]:= Plot[{f[x], fun[x]}, {x, 0, 1}]
```



Out[10]= - Graphics -

(4-8) $f(x) = x\sin x \qquad 0<x<\pi$

even over $-\pi<x<\pi$:

$$f(x) = \sum_{n=0}^{\infty} C_n \cos\frac{2n\pi}{2\pi}x$$

$$C_n = \frac{1}{\int_0^{\pi}\cos^2\frac{n\pi}{\pi}x\,dx}\int_0^{\pi} x\sin x\cos nx\,dx$$

$$\int_0^{\pi}\cos^2 nx\,dx = \frac{\pi}{2} \quad n\neq 0\,; \qquad = \pi \quad (n=0)$$

$n=0$ $\quad \int_0^{\pi} x\sin x\,dx = -x\cos x\Big|_0^{\pi} + \int_0^{\pi}\cos x\,dx = \pi$

$n\neq 0$ $\quad \int_0^{\pi} x\sin x\cos nx\,dx = \frac{1}{2}\int_0^{\pi} x\sin(n+1)x\,dx - \frac{1}{2}\int_0^{\pi} x\sin(n-1)x\,dx$

$\cancel{\int_0^{\pi} x\sin nx}$ $\quad \int_0^{\pi} x\sin(n+1)x\,dx = \frac{-1}{n+1}\pi(-1)^{n+1} + \frac{1}{n+1}\int_0^{\pi}\cos(n+1)x\,dx$

$$= \frac{\pi}{n+1}(-1)^n$$

$$\int_0^{\pi} x\sin(n-1)x\,dx = \frac{\pi}{n-1}(-1)^n \qquad (n\neq 1)$$

$$= 0 \qquad (n=1)$$

$$C_n = \begin{cases} 1 & n=0 \\ -\frac{1}{2} & n=1 \\ \frac{2}{(n^2-1)}(-1)^{n+1} & \text{otherwise} \end{cases}$$

odd: $f(x) = \sum C_n \cancel{\cos nx} \sin nx$

$$C_n = \frac{\int_0^\pi x \sin x \sin nx \, dx}{\int_0^\pi \sin^2 nx \, dx}$$

$$C_n = \frac{1}{\pi} \int_0^\pi x \left(\cos(n-1)x - \cos(n+1)x \right) dx$$

$n=1$ $$C_1 = \frac{1}{\pi} \int_0^\pi x (1 - \cos 2x) \, dx$$

$$= \frac{1}{\pi} \left[\frac{1}{2}\pi^2 - \int_0^\pi x \cos 2x \, dx \right]$$

$$= \frac{\pi}{2}$$

$n \neq 1$ $$C_n = \frac{1}{\pi} \left[\int_0^\pi x \cos(n-1) \, dx - \int_0^\pi x \cos(n+1)x \, dx \right]$$

$$\int_0^\pi x \cos(n-1)x \, dx = \frac{1}{n-1} x \sin(n-1)x \Big|_0^\pi - \frac{1}{n-1} \int_0^\pi \sin(n-1)x \, dx$$

$$= \frac{1}{(n-1)^2} \left((-1)^{n-1} - 1 \right) = \begin{cases} 0 & n \text{ odd} \\ -\frac{2}{(n-1)^2} & n \text{ even} \end{cases}$$

$$C_n = \begin{cases} 0 & n \text{ odd} \\ \frac{-2}{\pi(n-1)^2} + \frac{2}{\pi(n+1)^2} & n \text{ even} \end{cases}$$

$$C_n = \begin{cases} 0 & n \text{ odd} \\ -\frac{2}{\pi} \frac{4n}{(n-1)^2(n+1)^2} & n \text{ even} \end{cases}$$

odd is better expansion $C_n \sim \frac{1}{n^3}$