#1

```
In[1]:= f1 = x^2;
        f2 = Sin[x];
        f3 = Exp[x];

In[4]:= a11 = f1 * f1;
        a12 = f1 * f2;
        a13 = f1 * f3;
        a22 = f2 * f2;
        a23 = f2 * f3;
        a33 = f3 * f3;

In[20]:= m11 = (a11 /. x → 1.2) + (a11 /. x → 1.3) +
             (a11 /. x → 1.5) + (a11 /. x → 2.1) + (a11 /. x → 2.5);
         m12 = (a12 /. x → 1.2) + (a12 /. x → 1.3) + (a12 /. x → 1.5) +
             (a12 /. x → 2.1) + (a12 /. x → 2.5);
         m13 = (a13 /. x → 1.2) + (a13 /. x → 1.3) + (a13 /. x → 1.5) +
             (a13 /. x → 2.1) + (a13 /. x → 2.5);
         m22 = (a22 /. x → 1.2) + (a22 /. x → 1.3) + (a22 /. x → 1.5) +
             (a22 /. x → 2.1) + (a22 /. x → 2.5);
         m23 = (a23 /. x → 1.2) + (a23 /. x → 1.3) + (a23 /. x → 1.5) +
             (a23 /. x → 2.1) + (a23 /. x → 2.5);
         m33 = (a33 /. x → 1.2) + (a33 /. x → 1.3) + (a33 /. x → 1.5) +
             (a33 /. x → 2.1) + (a33 /. x → 2.5);

In[37]:= b1 = (0.25 * f1 /. x → 1.2) + (0.4 * f1 /. x → 1.3) +
             (0.6 * f1 /. x → 1.5) + (0.38 * f1 /. x → 2.1) + (-1.2 * f1 /. x → 2.5);
         b2 = (0.25 * f2 /. x → 1.2) + (0.4 * f2 /. x → 1.3) + (0.6 * f2 /. x → 1.5) +
             (0.38 * f2 /. x → 2.1) + (-1.2 * f2 /. x → 2.5);
         b3 = (0.25 * f3 /. x → 1.2) + (0.4 * f3 /. x → 1.3) + (0.6 * f3 /. x → 1.5) +
             (0.38 * f3 /. x → 2.1) + (-1.2 * f3 /. x → 2.5);

In[34]:= MatrixForm[mat = {{m11, m12, m13}, {m12, m22, m23}, {m13, m23, m33}}]
Out[34]//MatrixForm=
         ( 68.5028  12.7621  133.219 )
         ( 12.7621  3.89544  25.4405 )
         ( 133.219  25.4405  259.672 )

In[40]:= MatrixForm[c = {b1, b2, b3}]
Out[40]//MatrixForm=
         ( -3.4382  )
         ( 0.826783 )
         ( -6.52909 )

In[41]:= MatrixForm[Dot[Inverse[mat], c]]
Out[41]//MatrixForm=
         ( 2.19289  )
         ( 1.4979   )
         ( -1.29691 )
```

$g(x) = 2.19x^2 + 1.50\sin x - 1.30e^x$

#3

In[2]:=
```
r = {x, y, z};
MatrixForm[tensor = Dot[r, r] {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}} - Outer[Times, r, r]]
```

Out[3]//MatrixForm=

$$\begin{pmatrix} y^2+z^2 & -x\,y & -x\,z \\ -x\,y & x^2+z^2 & -y\,z \\ -x\,z & -y\,z & x^2+y^2 \end{pmatrix}$$

In[4]:=
```
MatrixForm[cyl = Integrate[
  ρ * tensor /. x → ρ Cos[θ] /. y → ρ Sin[θ], {z, -H/2, H/2}, {ρ, 0, R}, {θ, 0, 2 π}]]
```

Out[4]//MatrixForm=

$$\begin{pmatrix} \frac{1}{12} H \pi R^2 \left(H^2+3R^2\right) & 0 & 0 \\ 0 & \frac{1}{12} H \pi R^2 \left(H^2+3R^2\right) & 0 \\ 0 & 0 & \frac{1}{2} H \pi R^4 \end{pmatrix}$$

In[15]:=
```
density = M / (π R^2 H);
MatrixForm[mat = cyl * density]
```

Out[16]//MatrixForm=

$$\begin{pmatrix} \frac{1}{12} M \left(H^2+3R^2\right) & 0 & 0 \\ 0 & \frac{1}{12} M \left(H^2+3R^2\right) & 0 \\ 0 & 0 & \frac{M R^2}{2} \end{pmatrix}$$

In[5]:=
```
θ = ArcTan[H/2 / R];
ω = {Cos[θ], Sin[θ], 0}
```

Out[6]=

$$\left\{\frac{1}{\sqrt{1+\frac{H^2}{4R^2}}}, \frac{H}{2\sqrt{1+\frac{H^2}{4R^2}}\,R}, 0\right\}$$

In[17]:=
```
moment = FullSimplify[Dot[ω, Dot[mat, ω]]]
```

Out[17]=

$$\frac{1}{12} M \left(H^2+3R^2\right)$$

rotation from (θ_1, φ_1) to (θ_2, φ_2) #4

```
alpha = {{Cos[α], Sin[α], 0}, {-Sin[α], Cos[α], 0}, {0, 0, 1}};
beta = {{1, 0, 0}, {0, Cos[β], Sin[β]}, {0, -Sin[β], Cos[β]}};
```

In[71]:=
```
r = {Sin[θ] Cos[ϕ], Sin[θ] Sin[ϕ], Cos[θ]};
```

In[5]:=
```
θ1 = 15/180 π; ϕ1 = 150/180 π; θ2 = 75/180 π; ϕ2 = -60/180 π;
```

In[6]:=
```
R = Dot[Transpose[alpha /. α -> ϕ2 - π/2],
  Dot[Transpose[beta /. β -> θ1 - θ2], Transpose[alpha /. α -> π/2 - ϕ1]]];
```

In[63]:=
```
N[r /. θ -> θ2 /. ϕ -> ϕ2]
N[Dot[R, r /. θ -> θ1 /. ϕ -> ϕ1]]
```

Out[63]= {0.482963, -0.836516, 0.258819}

Out[64]= {0.482963, -0.836516, 0.258819} > SAME

In[65]:=
```
(* or *)
r0 = Dot[Transpose[beta], N[r /. θ -> θ1 /. ϕ -> ϕ1]];
Dot[{0, 0, 1}, r0]
```

just $(alpha)^T \cdot (beta)^T$

Out[65]= 0.965926 Cos[β] + 0.12941 Sin[β]

In[66]:=
```
Solve[0.965925826289067934` Cos[β] + 0.129409522551260325` Sin[β] == Cos[θ2], β]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found.

Out[66]= {{β → -1.16881}, {β → 1.43518}}

In[67]:=
```
r0 = Dot[Transpose[beta /. β -> -1.16881], N[r /. θ -> θ1 /. ϕ -> ϕ1]]
```

intermediate r_0

Out[67]= {-0.224144, 0.939559, 0.258822}

In[68]:=
```
(* x-component negative *)
ϕ0 = ArcTan[N[Dot[{0, 1, 0}, r0] / Dot[{1, 0, 0}, r0]]] + π;
```

In[69]:=
```
R1 = N[Dot[Transpose[alpha /. α -> ϕ2 - ϕ0], Transpose[beta /. β -> -1.16881]]];
```

In[70]:=
```
Dot[R1, r /. θ -> θ1 /. ϕ -> ϕ1]
```

Out[70]= {0.482963, -0.836516, 0.258822} ✓

In[62]:=
```
MatrixForm[R - R1]
```

Out[62]//MatrixForm=

$$\begin{pmatrix} 0.308892 & -0.736658 & 0.170373 \\ 0.41039 & -0.274543 & 0.132012 \\ 0.75 & 0.487273 & 0.108753 \end{pmatrix}$$

$R \neq R1$