# Mid term solution

① when $v = \frac{dx}{dt} = \alpha\sqrt{x}$ what is $x(t)$ ?

$$\frac{d^2x}{dt^2} = \alpha \frac{1}{2\sqrt{x}}\frac{dx}{dt} = \frac{\alpha}{2}\frac{\alpha\sqrt{x}}{\sqrt{x}} = \frac{\alpha^2}{2} = \text{constant}$$

therefore $\frac{dx}{dt} = \frac{\alpha^2}{2} t + C_1$ ; $x = \frac{\alpha^2}{4} t^2 + C_1 t + C_2$

$x(0) = 0 \quad C_2 = 0$

$$v^2 = \alpha^2 x \Rightarrow \left(\frac{\alpha^2}{2} t + C_1\right)^2 = \alpha^2\left(\frac{\alpha^2}{4} t^2 + C_1 t\right)$$

$$\frac{\alpha^4}{4} t^2 + \alpha^2 t C_1 + C_1^2 = \frac{\alpha^4}{4} t^2 + \alpha^2 C_1 t \ ; \quad C_1 = 0$$

$$x(t) = \frac{\alpha^2}{4} t^2 \quad *$$

② $4^{th}$ degree polynomial 5 constants

leading coefficient equal 1.0, 4 constant

$$f(x) = x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

since $f(x) - x = 0$ at $x = 2, 3, 4, 5$

$$f(x) - x = (x-2)(x-3)(x-4)(x-5)$$

therefore $f(1) = (-1)(-2)(-3)(-4) + 1 = 25$

$a_3 = -2-3-4-5 = -14$

$a_2 = 6+8+10+12+15+20 = 71$

$a_1 = -24-30-40-60+1 = -153$

$a_0 = 120$

(4) $T = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) \qquad V = -\frac{k}{r}$

$$T - V = \mathcal{L} \qquad p_\theta = \frac{\partial(\mathcal{L})}{\partial\dot{\theta}} = mr^2\dot{\theta}$$

transformed $\tilde{\mathcal{L}} = mr^2\dot{\theta}\dot{\theta} - \mathcal{L}$

$$\tilde{\mathcal{L}} = -\frac{m}{2}\dot{r}^2 - \frac{k}{r} + \frac{m}{2}r^2\dot{\theta}^2$$

$$\tilde{\mathcal{L}} = -\frac{m}{2}\dot{r}^2 + \frac{p_\theta^2}{2mr^2} - \frac{k}{r} = -\tilde{T} + \tilde{V}$$

(Lagrangian arbitrary constant)

effective potential $= \frac{p_\theta^2}{2mr^2} - \frac{k}{r}$

(5) $H = U + PV$

$$\left.\frac{\partial H}{\partial p}\right)_T = \left.\frac{\partial U}{\partial p}\right)_T + V + P\left.\frac{\partial V}{\partial p}\right)_T$$

since $\left.\frac{\partial U}{\partial p}\right)_T = \left.\frac{\partial U}{\partial V}\right)_T \left.\frac{\partial V}{\partial p}\right)_T = \left.\frac{\partial V}{\partial p}\right)_T \left[T\left.\frac{\partial P}{\partial T}\right)_V - P\right]$

$$\left.\frac{\partial H}{\partial p}\right)_T = T\left.\frac{\partial V}{\partial p}\right)_T \left.\frac{\partial P}{\partial T}\right)_V - \cancel{P\left.\frac{\partial V}{\partial p}\right)_T} + V + \cancel{P\left.\frac{\partial V}{\partial p}\right)_T}$$

so $\left.\frac{\partial H}{\partial p}\right)_T = T\left.\frac{\partial V}{\partial p}\right)_T \left.\frac{\partial P}{\partial T}\right)_V + V$

or: $\left.\frac{\partial H}{\partial p}\right)_T = V - T\left.\frac{\partial V}{\partial T}\right)_P$

(6) $Pr(P/D) = 0.9$ , $Pr(P/\bar{D}) = 0.05$ , $Pr(D) = 10^{-4}$

a) therefore $Pr(D/P) = \dfrac{10^{-4} * 0.9}{10^{-4} * 0.9 + (1-10^{-4}) * 0.05}$

$$Pr(D/P) \simeq \frac{0.9}{0.9 + 10^{4} * 0.05} = 1.8 \times 10^{-3}$$

b) $$Pr(\bar{D}/\bar{P}) = \frac{Pr(\bar{D})(1 - Pr(P/\bar{D}))}{Pr(\bar{D})(1 - Pr(P/\bar{D})) + Pr(D)\, Pr(\bar{P}/D)}$$

$$Pr(\bar{D}/\bar{P}) \simeq \frac{1 * 0.95}{1 * 0.95 + 10^{-4}(1-0.9)} = 1$$

c) reduce $Pr(P/\bar{D})$, reduce false positive

(7) all ravens are black $Pr(B/R) = 1$

$$Pr(B/R) = \frac{Pr(R \cap B)}{Pr(R)}$$

because $R = R \cap (B \cup \bar{B}) = (R \cap B) \cup (R \cap \bar{B})$

$$Pr(R) = Pr(R \cap B) + Pr(R \cap \bar{B}) + Pr(R \cap B \cap R \cap \bar{B})$$

$$R \cap B \cap R \cap \bar{B} = (B \cap \bar{B}) \cap R = \emptyset \quad Pr(\emptyset) = 0$$

$$Pr(B/R) = 1 \Rightarrow Pr(\bar{B} \cap R) = 0 \Rightarrow \begin{matrix} R = \emptyset & \times \\ \bar{B} = \emptyset & \checkmark \end{matrix}$$

But $Pr(\bar{R}/\bar{B}) = 1$ implies $Pr(\bar{B} \cap R) = 0 \Rightarrow \begin{matrix} \bar{B} = \emptyset & \times \\ R = \emptyset & \checkmark \end{matrix}$

so $Pr(B/R) = 1$ include $\bar{B} = \emptyset$ cases

$Pr(\bar{R}/\bar{B}) = 1$ include $R = \emptyset$ cases

Not equivalent !