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ON THE CHOICE OF BOUNDARY CONDITIONS IN CONTINUUM MODELS OF CONTINENTAL DEFORMATION

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Abstract. Recent studies of continental deformation have treated the lithosphere as a viscous media and investigated the time evolution of the deformation caused by tectonic and buoyancy forces. We examine the differences between continuum models that keep velocity boundary conditions (b.c.) constant with time and models that keep stress b.c. constant with time and demonstrate these differences by using a simple example of a continental lithosphere that is subjected to horizontal compression. Our results show that in the case of constant stress b.c., the indentation velocity decreases with time, while in the case of constant velocity b.c., the indentation velocity remains constant with time. Observations from the Tibetan plateau and from the Andes indicate that the rate of indentation decreases with time. This suggests that when buoyancy forces are comparable in magnitude to tectonic forces, constant stress b.c. are more appropriate for time evolutionary models of continental lithosphere. We propose a simple procedure that translates velocity b.c., which are easy to estimate, into stress b.c. at the initial stage, and keep the stress b.c. constant along the time progression of the calculations.

Introduction

In recent years, continuum models of viscous media have been widely used to investigate large scale continental deformation [e.g., Bird and Piper, 1980; England and McKenzie, 1982; Vilotte et al., 1982]. Application of these models to real situations requires realistic estimates for the boundary conditions (b.c.), which are derived from various observations. Studies of marine magnetic anomalies and geological observations, which describe the kinematics of the earth surface, yield reliable estimates of velocity b.c. Less reliable estimates of stress b.c. can be derived from observations of active deformation (seismic activity or geodetic measurements) and from models of plate driving forces. Thus, practical considerations favor the choice of velocity b.c., which are widely used.

Investigation of the time history of the deformation in response to a tectonic process, (e.g., the indentation of India into Eurasia) requires specification of b.c. at various times as the deformation evolves. Because there are few constraints on the change of the b.c. with time, it is easiest to assume that the b.c. remain constant with time. The purpose of this study is to compare and to evaluate the differences between models that keep velocity b.c. constant with time and models that keep stress b.c. constant with time, and to assess which might be more realistic or appropriate.

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The Forces that Dominate Continental Deformation

The continental lithosphere, as any continuum, is subjected to surface and body (volume) forces. Surface forces act on the boundaries of the lithosphere and give rise to stresses that can be described by a pressure term and a deviatoric stress tensor. The pressure (to first order) balances the weight of the rock column and does not cause any lithospheric deformation. Deviatoric stresses arise from tectonic forces that act on the lithosphere's boundaries and deform the lithosphere. Tectonic forces that are most often used in continuum models of continental lithosphere are horizontal forces acting on plate boundaries and asthenospheric shear tractions acting on the base of the lithosphere. Body forces act on the mass of each element within the lithosphere, are due to gravity, and are often called buoyancy forces. Although buoyancy forces act vertically, horizontal density variations will induce horizontal pressure gradients, which can drive horizontal flow. Isostatically-compensated variations in the crustal thickness are the simplest form of horizontal density variations and are the most common source of buoyancy forces in continuum models.

In regions of active tectonism, the effect of the tectonic and the buoyancy forces may change with time. We demonstrate these changes via a simple example of continental lithosphere that is subjected to horizontal compression. Figure 1 shows a schematic model of a continental lithosphere comprised of buoyant crust overlying a heavy mantle. One of its side boundaries is being indented by a tectonic force. At the initial stage the crust is assumed to be of uniform thickness, thus the contribution of the buoyancy forces are zero (Figure 1*a*). At a later stage, the lithosphere has shortened and the continental crust has thickened (Figure 1*b*). The thickening of the crust near the indented boundary gives rise to a buoyancy force, which resists the indentation caused by the tectonic force.

The motion of the indenting boundary is proportional to the viscous force that balance the net force (tectonic + buoyancy) acting on the boundary. Hence, the indentation velocity is proportional to the net force acting on the indenting boundary. When the tectonic and the buoyancy forces acting on the indenting boundary are equal and opposite, the net force is zero and the indenting boundary does not move. However, if the net force acting on the indenting boundary is directed inland the boundary moves and thickens the crust. If the net force is opposite in direction the boundary moves outward releasing the potential energy stored in the thick crust.

Boundary Conditions

Both velocity and (deviatoric) stress b.c. can be used in viscous flow models of the continental lithosphere to investigate the indentation produced by the above example.

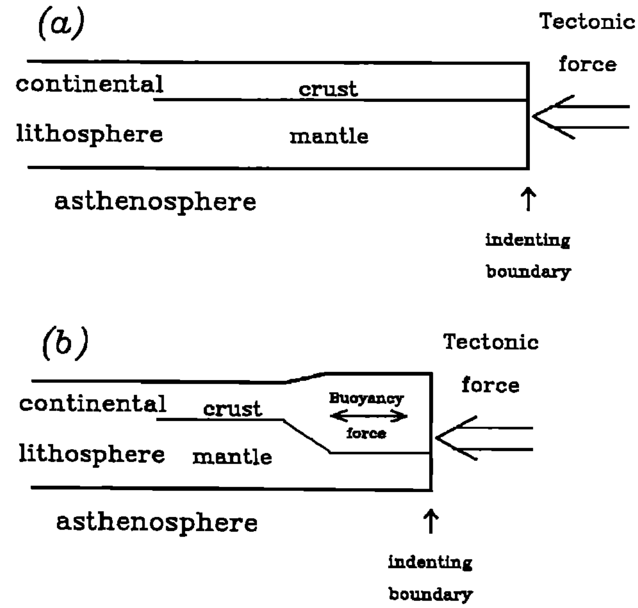

Fig. 1. Schematic diagram of the model showing a continental lithosphere that is subjected to horizontal compression by a tectonic force. At the initial stage (*a*), the buoyancy forces are zero because the thickness of the crust is assumed to be uniform. At a later stage (*b*), the crust has been thickened by horizontal compression, which gives rise to a buoyancy force. The compressional tectonic force and the buoyancy force act on the indenting boundary in opposite directions.

Velocity b.c., which determine the motion of the indenting boundary, are proportional to the net force acting on the indenting boundary, whereas, deviatoric stress b.c. represent only the tectonic force acting on that boundary. We examine the differences between constant (with time) velocity and constant stress b.c. when they are applied to the above example. Constant velocity b.c. implies that the indentation velocity, and hence, the net force (tectonic + buoyancy) are kept constant with time. Constant stress b.c. indicate that the force per unit area acting on the indenting boundary is kept constant with time. Because the change in the lithospheric thickness is small with respect to the total thickness of the lithosphere (5 km vs. 100 km), the area of the indenting boundary approximately remains constant with time. Therefore, constant stress b.c. implies that only the tectonic force is kept constant with time.

The thickening of the crust due to the indentation increases the effect of the buoyancy force, which acts on the indenting boundary in the opposite direction to that of the tectonic force. In the case of constant stress b.c., the magnitude of the tectonic force remains constant with time, while the magnitude of the buoyancy force increases with time. As a result, the net force acting on the boundary and, hence, the indentation velocity decrease with time (Figure 2*a*). In the case of constant velocity b.c., the indenting velocity and, hence, the net force acting on that boundary are kept constant with time. Initially, when the buoyancy force is zero, there is no difference between the two models. The thickening of the crust with time increases the effect of the buoyancy force, as in the case of constant stress b.c. Because the net force that acts on the indenting boundary remains constant, the effect of the tectonic force must increase with time in order to balance the increasing effect of the buoyancy force (Figure 2*b*).

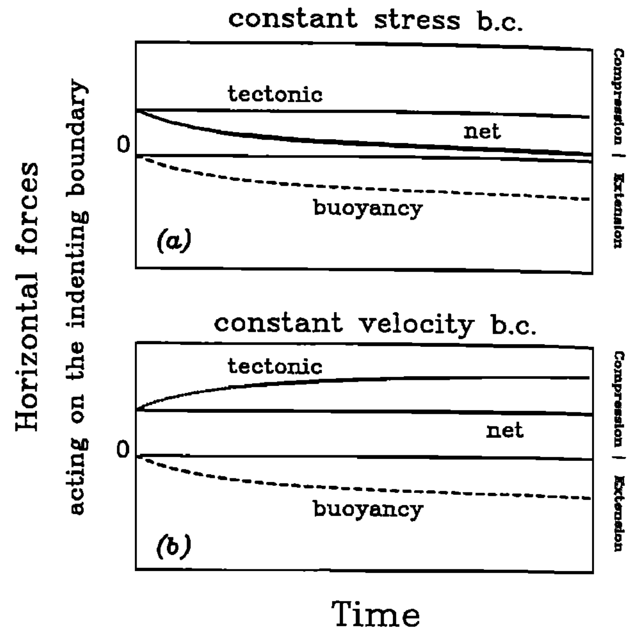

Fig. 2. Schematic plot of the horizontal forces acting on the indenting boundary of the continental lithosphere, as it progress with time. The velocity of the indenting boundary is proportional to the net force acting on the boundary (see text).
(*a*) Constant stress boundary conditions: the magnitude of the tectonic force remains approximately constant, but the magnitude of the buoyancy force increases as the crust thickens. The magnitude of the net force (tectonic + buoyancy) and, hence, of the indentation velocity decrease with time. (*b*) Constant velocity boundary conditions: the magnitude of the net force remains constant with time and the magnitude of the buoyancy force increases as the crust thickens. In order to keep the constant velocity condition the magnitude of the tectonic force increases with time to balance the increasing magnitude of the buoyancy force.

Discussion

If buoyancy forces are negligible, the net force represents mainly the effect of tectonic forces and differences between calculations with constant velocity b.c. and constant stress b.c. are negligible as well. England and Jackson [1989] estimate that buoyancy forces acting on large elevated plateaux are comparable in magnitude to tectonic forces associated with plate motion. Thus, deformation in regions of high elevated plateaux represents the combined effect of the tectonic and the buoyancy forces. The simple example we use above can demonstrate the importance of constant stress b.c. in continuum models of continental deformation. If the indentation velocity remains constant with time, constant velocity b.c. are sufficient. However, if the indentation velocity decreases with time, we should favor the constant stress b.c. In this simple test we assume that any change in the indentation velocity is a result of local force balance between a constant tectonic force and increasing effect of the buoyancy force. Changes in global force balance may change the magnitude of the tectonic force and bias our results. Therefore, we use observations from two regions of compressional tectonics, the Tibetan plateau and the Andes, to examine the above predictions and to look for consistency in our results.

The indentation of the Indian plate into Eurasia shortened and thickened the crust underneath Tibet during the past 40–50 m.y., giving rise to the uplift of the Tibetan plateau. The 2000 km of horizontal indentation, as indicated from studies of marine magnetic anomalies and from geological observations, suggests an averaged indentation velocity of 40–50 mm yr^{-1} [England and McKenzie, 1982]. However, the seismic activity in Central Asia indicates a slower indentation rate of about 20 mm yr^{-1} [Molnar and Deng, 1984]. England and McKenzie [1982] use a constant indentation velocity of 50 mm yr^{-1} to calculate the formation of the thick Tibetan crust and the high plateau. Their model failed to explain the present-day E–W extension observed in the Tibetan plateau, which can be explained by a slower rate of indentation during the past 5 m.y. [England and Houseman, 1989]. In the Andes, geological observations suggest an average shortening rate of about 10 mm yr^{-1} for the past 15–25 m.y. [Isacks, 1988]. However, seismic observations show a much slower current shortening rate, about 2 mm yr^{-1} [Suárez, 1983]. Wdowinski and O'Connell [1990] explain this difference by the decrease of the indentation velocity owing to the increasing effect of buoyancy forces. In both regions the short term rate, which is derived from seismic activity, is significantly lower than the long term geologically observed rate. Because seismic deformation accounts for only 25–70% of the total deformation [Ekström and England, 1990], the present-day shortening rate should be higher than the seismically observed rate. However, models of the deformation in the Tibetan plateau and in the Andes indicate that the present-day shortening rate is lower than the long term averaged rate.

The decreasing rate of horizontal shortening observed in the Tibetan plateau and in the Andes suggests that constant stress b.c. are more appropriate to use in continuum models of continental deformation in a compressional tectonic environment, where buoyancy forces are comparable in magnitude to tectonic forces. In an extensional tectonic environment, crustal thinning gives rise to buoyancy forces that resist the extension. Because regions with a significantly thin crust are limited in area - much less than areas of high elevated plateaux - the magnitude of buoyancy forces are probably less than of tectonic forces. In a pure transcurrent environment there is no change in the crustal thickness and buoyancy forces are negligible. In reality, along strike-slip faults there are local regions of hybird deformation (extension or compression) that change the thickness of the crust and give rise to buoyancy forces. As a result, buoyancy forces may affect the deformation locally. In summary, constant stress b.c. are more appropriate to use in models of continental deformation, although models that use constant velocity b.c. will probably generate similar result in transcurrent and extensional tectonic environments.

Our analysis suggests that constant stress b.c. are more appropriate to use in continuum models of continental deformation. Because stress b.c. are more difficult to reliably estimate, we suggest the following procedure. At the initial stage, the deformation can be calculated by using the velocity b.c., which can give an initial estimate of the horizontal stresses acting on the indenting boundary. The effect of the buoyancy force at the initial stage is negligible; thus, the stresses that are evaluated along the indenting boundary represent the tectonic force. At following time steps, these stresses can be used as stress b.c. along the indenting boundary. Finally, the solution can be scaled by an independent observation, such as total indentation, which enables evaluation of the appropriate initial indentation velocity and stress b.c.

Conclusions

Continuum models are used to investigate large scale continental deformation in response to velocity or stress b.c. We examine the effect of constant (with time) velocity versus constant stress b.c. in a simple example of continental lithosphere that is subjected to horizontal compression. The velocity of the indenting boundary is proportional to the net force (tectonic + buoyancy) acting on the boundary. In the case of constant stress b.c., the effect of buoyancy force increases with time whereas the effect of the tectonic force remains approximately constant; thus, the net force acting on the boundary and the indentation velocity decrease with time. For constant velocity b.c., the net force acting on the boundary remains constant with time. Thus, the effect of the tectonic force must increase with time in order to balance the increasing effect of the buoyancy force as the crust thickens. The decreasing rate of horizontal shortening observed in the Tibetan plateau and in the Andes suggests that constant stress b.c. are more appropriate for investigation of large scale continental deformation.

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