MAS 3105 (LINEAR ALGEBRA)
Assignment 1, due Wednesday May 20, 2015

Name: **PID:**

Remember that you won't get any credit if you do not show the steps to your answers. No late assignment will be accepted.

1. Let $m,\ a,\ b,\ c,\ d$ be real numbers. Consider the linear system

$$\begin{cases} x+(m+1)y+2mw=a \\ mx+z+w=b \\ (2m+1)x+y+(m+1)z+w=c \\ (m+1)z+(m+1)w=d. \end{cases}$$

a) Write down the matrix A_m corresponding to this system.
b) Find all values of m for which A_m is singular. (Hint. Find the determinant of A_m.)
c) For each value obtained in b), find necessary and sufficient conditions on a, b, c, and d such that the linear system is consistent, and find all solutions in each case.

2) Let A be an $n\times n$ matrix satisfying $I_n + A - 5A^2 + 7A^5 - 9A^{11} = 0_{\mathcal{M}_n}$. Show that A is nonsingular, and find its inverse.

3) Consider the matrices

$$A=\begin{pmatrix} 1 & -1 & 2 \\ 3 & -2 & 1 \\ 4 & -3 & 5 \end{pmatrix},\quad B=\begin{pmatrix} 1 & -1 & 2 \\ 3 & -2 & 1 \\ 6 & -5 & 9 \end{pmatrix},\quad C=\begin{pmatrix} -3 & -1 & 2 \\ 1 & -2 & 1 \\ -6 & -3 & 5 \end{pmatrix}$$

Find an elementary matrix E such that $EA=B$ and an elementary matrix F such that $AF=C$.

4) Let A, B, and C be $n\times n$ matrices such that $A\neq 0_{\mathcal{M}_n}$, $B\neq 0_{\mathcal{M}_n}$ $C\neq 0_{\mathcal{M}_n}$, and $ABC=0_{\mathcal{M}_n}$. Show that at least two of those matrices are singular.