

MAS 3105 (LINEAR ALGEBRA)
Assignment 1, due Wednesday May 25, 2016

Remember that you won't get any credit if you do not show the steps to your answers. Neither me nor the LA can help you with this assignment, as it is graded. Assignments written the same way will all get zero. No late assignment will be accepted.

Name:

PID:

Remember that you won't get any credit if you do not show the steps to your answers. No late assignment will be accepted.

1. Let m be a real number. Consider the linear system

$$\begin{cases} x + 2y + 3z + mw = m - 1 \\ 2x + y + mz + 3w = 1 \\ 3x + my + z + 2w = 0 \\ mx + 3y + 2z + w = 0. \end{cases}$$

- a) Write down the matrix A_m corresponding to this system.
 - b) Find all values of m for which A_m is singular. (Hint. You may first start the Gauss reduction process placing three zeros on the first column, then two zeros on the second column, then find the determinant of the lower 2×2 sub-matrix of A_m , and set it equal to zero.)
 - c) For each value obtained in b), find all possible solutions.
- 2) Let A and B be $n \times n$ matrices such that A is nonsingular and $B^2 = 0_{\mathcal{M}_n}$. Set $K = I_n + A^{-1}BA$. Show that K is nonsingular, and find its inverse. (Hint. Find K^2 .)
- 3) Consider the matrices

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & -1 \\ -1 & 2 & 0 \end{pmatrix}.$$

Show that B is invertible, and compute a matrix C such that $A = BC$.

4) Let A be an $m \times n$ matrix having a row consisting entirely of zeros. Let B be an $n \times r$ matrix. Show that the matrix AB has a row consisting entirely of zeros. (Hint. Suppose that the i th row of A is the one consisting entirely of zeros and use the definition of the product of two matrices to find the row of AB consisting of zeros only.)

Note. An $n \times n$ matrix A is called nonsingular if there exists an $n \times n$ matrix A_1 such that $AA_1 = A_1A = I_n$, where I_n is the $n \times n$ identity matrix, that is, a matrix having only ones on its diagonal, and zero everywhere else. When A is nonsingular, we say that A is invertible, and write $A_1 = A^{-1}$.