MAS 3105 (LINEAR ALGEBRA)
Assignment 2, due Wednesday June 03, 2015

Name: **PID:**

Remember that you won't get any credit if you do not show the steps to your answers. Neither me nor the LA can help you with this assignment, as it is graded. Assignments written the same way will all get zero. No late assignment will be accepted.

1. For which values of the parameter m are the vectors $\mathbf{u}_1 = (2, 1, 3)^T$, $\mathbf{u}_2 = (1, m, 1)^T$ and $\mathbf{u}_3 = (-1, 1, -m)^T$ linearly independent?

2) In $\mathbb{R}^5$, consider the vectors $\mathbf{v}_1 = (1, 2, -4, 3, 1)^T$, $\mathbf{v}_2 = (2, 5, -3, 4, 8)^T$, $\mathbf{v}_3 = (6, 17, -7, 10, 22)^T$, $\mathbf{v}_4 = (1, 3, -3, 2, 0)^T$. Find real numbers a and b such that the vector $\mathbf{v} = (2, 4, 6, a, b)$ lies in $\text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4)$.

3) Let $\mathbf{u}_1 = (1, 2, 3)^T$, $\mathbf{u}_2 = (1, -2, 1)^T$, $\mathbf{u}_3 = (2, 2, 1)^T$ be vectors in $\mathbb{R}^3$. Consider the vectors $\mathbf{v}_1 = (1, 2, 1)^T$, $\mathbf{v}_2 = (5, -3, 4)^T$, $\mathbf{v}_3 = (1, 1, 1)^T$.

   a) Show that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ and $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are bases of $\mathbb{R}^3$.

   b) Find the transition matrix from the ordered basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ to the ordered basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

   c) If $\mathbf{u} = 5\mathbf{u}_1 + 2\mathbf{u}_2 - 4\mathbf{u}_3$, find the coordinates of $\mathbf{u}$ in the ordered basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

4) Let A and B be $n \times n$ matrices, and set $R(A) = \{\mathbf{z} \in \mathbb{R}^n ; \mathbf{z} = A\mathbf{x} \text{ for some } \mathbf{x} \in \mathbb{R}^n\}$=Column Space of A.

   a) Show that $R(A + B) \subseteq R(A) + R(B)$.

   b) Suppose that $AB = \mathbf{0}_{\mathcal{M}_n}$, and $A + B$ is nonsingular. Show that $r_A + r_B = n$, where r_A and r_B stand for rank of A and rank of B respectively. (Hint. You may use the rank-nullity theorem.)