MAS 3105 (LINEAR ALGEBRA)
Assignment 3, due Wednesday June 22, 2016

Name: **PID:**

Remember that you won't get any credit if you do not show the steps to your answers. Neither me nor the LA can help you with this assignment, as it is graded. Assignments written the same way will all get zero. Do your best. No late assignment will be accepted.

1. Let $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$.

a) Find all the eigenvalues of A.

b) For each eigenvalue, find the corresponding eigenspace.

c) Find a diagonalizing matrix U for A.

d) Find a matrix B such that $B^5 = A$.

2) On $C^1([0,1])$, define a mapping N by $N(f) = \left(|f(0)|^2 + \int_0^1 |f'(t)|^2 \, dt \right)^{\frac{1}{2}}$.

a) Show that N is a norm on $C^1([0,1])$.

b) Find $N(g)$ if $g(x) = \cos(\pi x)$.

c) Consider the inner product associated to N given by $\langle f, g \rangle = f(0)g(0) + \int_0^1 f'(t)g'(t) \, dt$.
Let $p(x) = \cos(\pi x)$ and $q(x) = 2x + 1$.
c1) Find the angle θ between p and q.
c2) Find the best least squares approximation to $h(x) = e^{-x}$ by a function $w(x) = ap(x) + bq(x)$, where a and b are constants to be determined.

3) Let $M = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 2 & 1 & 1 & 2 \\ -1 & 3 & 1 & 4 \\ 1 & 1 & -1 & 2 \end{pmatrix}$.

a) Find an orthonormal basis for the column space of M.

b) Find the QR factorization for M, where the column vectors of Q form an orthonormal basis for the column space of M.

c) Solve the least squares problem $M\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = (-1, 1, 2, 3)^T$.

4) Let A be an $m \times n$ matrix with rank n. Set $P = A(A^T A)^{-1} A^T$.

a) Show that $P^2 = P$.

b) Show that $P^r = P$, for all integers $r \geq 2$.

c) Define the exponential of P by setting $e^P = \sum_{k=0}^{\infty} \frac{P^k}{k!}$. Find e^{2P}, and express it without the summation symbol.

5) Find the best least squares fit by a linear polynomial to the data in the chart.

x	2	4	5	6
y	1	2	3	3