Exercises set 1 - Sum 15

A Solve each system

1.
$$\begin{aligned} x_1 + 2x_2 &= 5 \\ x_1 - x_2 &= 2 \\ -x_1 + 3x_2 &= 3 \end{aligned}$$

2.
$$\begin{aligned} x_2 + x_3 + x_4 &= 0 \\ 3x_1 \qquad + 3x_3 - 4x_4 &= 7 \\ x_1 + x_2 + x_3 + 2x_4 &= 6 \\ 2x_1 + 3x_2 + x_3 + 3x_4 &= 6 \end{aligned}$$

B. Solve the following system according to the values of the parameter m.

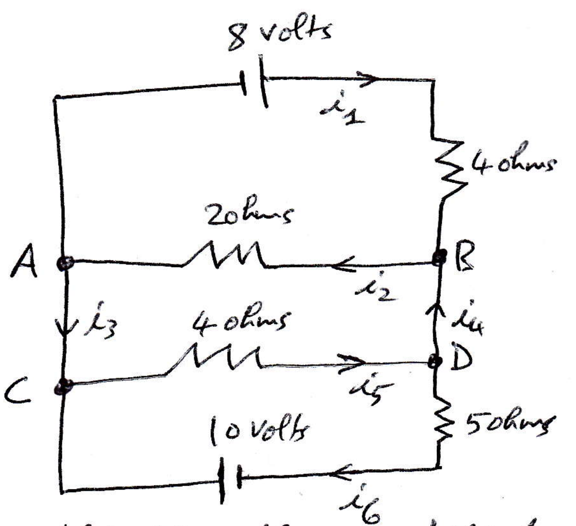
$$\begin{cases} 2x_1 - x_2 + 3x_3 = 0 \\ -4x_1 + mx_2 + x_3 = 0 \\ -2x_1 + x_2 + mx_3 = 0 \\ 10x_1 - 5x_2 - 6x_3 = 0 \end{cases}$$

C. Find the reduced row echelon form of the matrix according to the values of m.

$$A_m = \begin{pmatrix} 1 & m+1 & 0 & 2m \\ m & 0 & 1 & 1 \\ 2m+1 & 1 & m+1 & 1 \\ 0 & 0 & m+1 & m+1 \end{pmatrix}$$

Hint. Discuss the case $m=0$, then $m=-1$, and finally the case $m=1$.

D. Consider the following network:



Write down the equations satisfied by the currents $i_1, i_2, \dots, i_6$.

Kirchhoff Laws

i) At every node, the sum of the incoming currents equals the sum of the outgoing currents.

ii) Around every closed loop, the algebraic sum of the voltage gains must equal the algebraic sum of the voltage drops.

ohm's Law: The voltage drop E for each resistor is given by $E = iR$, where i stands for the current in amperes and R the resistance in ohms.

E.

1) Let A be an $m \times n$ matrix. Show that $A^T A$ and $A A^T$ are both symmetric.

2) An $n \times n$ matrix A is called skew symmetric if $A^T = -A$. Show that if A is a skew symmetric matrix, then its diagonal entries must all be zeros.

3) Let A and B be symmetric $n \times n$ matrices. Prove that $AB = BA$ if and only if AB is also symmetric.

4) Prove that if a matrix A is nonsingular, then A^T is nonsingular, and $(A^T)^{-1} = (A^{-1})^T$.

5) Let A be a nonsingular $n \times n$ matrix. Use mathematical induction to prove that A^m is nonsingular and $(A^m)^{-1} = (A^{-1})^m$, $m = 1, 2, 3, \ldots$

6. Let $p \geq 2$ be an integer. A matrix A is called nilpotent of order p if $A^p = O_{M_n}$ but $A^{p-1} \neq O_{M_n}$. Let A be a nilpotent matrix of order p. Show that $I_n - A$ is nonsingular and

$$(I_n - A)^{-1} = I + A + A^2 + \cdots + A^{p-1}.$$