Problem Set 2 - MAS3105 - SUM 15.

1) Show that if A is nonsingular, then $\operatorname{adj}A$ is nonsingular, and $(\operatorname{adj}A)^{-1} = \operatorname{adj}(A^{-1})$

2) Let $u_1, u_2, \dots, u_n$ be linearly independent vectors in a vector space E, and $v = \sum_{j=1}^{n} \alpha_j u_j = \alpha_1 u_1 + \alpha_2 u_2 + \dots + \alpha_n u_n$, $\alpha_1, \alpha_2, \dots, \alpha_n$ are scalars. For each $j = 1, 2, \dots, n$, set $w_j = u_j + v$. Show that $w_1, w_2, \dots, w_n$ are linearly independent if and only if $\sum_{k=1}^{n} \alpha_k \neq -1$.

3) Let u_1, u_2, u_3 be in $\mathbb{R}^3$ such that $\mathbb{R}^3 = \operatorname{Span}(u_1, u_2, u_3)$. Do we have $\mathbb{R}^3 = \operatorname{Span}(u_1, u_1+u_2, u_1+u_2+u_3)$?

4) Let U, V, W be subspaces of a vector space E. Show that

a) $\operatorname{Span}(U \cup V) = U + V$. (Remember $U+V = \{z \in E;\ z = u+v,\ u \in U, v \in V\}$.)

b) $(U \cap V) + (U \cap W) \subseteq U \cap (V+W)$

c) $U + (V \cap W) \subseteq (U+V) \cap (U+W)$

d) $U \subseteq V \Rightarrow (V \cap (U+W) = (V \cap U) + (V \cap W))$

e) $V \subseteq U \Rightarrow V + (U \cap W) = U \cap (V+W)$

(Hint. To show that two sets are equal, say $A=B$, show $A \subseteq B$ and $B \subseteq A$.)

5) Let $u_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$, $u_2 = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 6 \end{pmatrix}$, $u_3 = \begin{pmatrix} 0 \\ 2 \\ 4 \\ 4 \end{pmatrix}$, $u_4 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 2 \end{pmatrix}$; $u_5 = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 1 \end{pmatrix}$

Do u_1, u_2, u_3, u_4 span $\mathbb{R}^4$? What about u_1, u_2, u_3 and u_5?

6) Let u_1, u_2, u_3 be linearly independent vectors in a vector space E. Set $v_1 = u_1 - 2u_2$, $v_2 = 2u_2 - 3u_3$, $v_3 = 3u_3 - 4u_1$.

Are v_1, v_2, v_3 linearly independent?