1. Find all least squares solutions of $Ax = b$ if

   a) $A = \begin{pmatrix} 1 & 1 & 3 \\ -1 & 3 & 1 \\ 1 & 2 & 4 \end{pmatrix}$ and $b = \begin{pmatrix} -2 \\ 0 \\ 8 \end{pmatrix}$.

   b) $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

2. Find the best least squares fit by a quadratic polynomial to the data

x	-1	0	1	2
y	0	1	3	9

3. On $C([0,1])$, define the inner product $\langle f, g \rangle = \int_0^1 f(x) g(x) dx$.
   Let S be the subspace spanned by the vectors 1 and $2x - 1$.

   a) Show that 1 and $2x - 1$ are orthogonal.

   b) Determine $\|1\|$ and $\|2x - 1\|$.

   c) Find the best least squares approximation to $\sqrt[4]{x}$ by a function from S.

4. On M_2, consider the inner product $\langle A, B \rangle = aa' + bb' + cc' + dd'$ if
   $A = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$ and $B = \begin{pmatrix} a' & c' \\ b' & d' \end{pmatrix}$. Let $C = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$. Find a matrix D
   such that $\langle C, D \rangle = 0$. What is $\|C\|$? $\|D\|$?

5. Let $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \\ 1 & 2 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$

   a) Use the Gram-Schmidt process to find an orthonormal basis for $R(A)$.

   b) Find a QR factorization of A

   c) Solve the least squares problem $Ax = b$.

6. Let A, B be $n \times n$ matrices. Show that if none of the eigenvalues of A is equal to 1, then the matrix equation $XA + B = X$, $(X \in M_n)$ has a unique solution.

7. Let $A \in M_n$, and let d be a nonzero eigenvalue of A. Let x be an eigenvector corresponding to d. Show that x lies in the column space of A. Where would x lie if $d = 0$?

8. Let $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ 3 & 6 & -3 \end{pmatrix}$. Find a matrix D that is diagonal and a nonsingular matrix X such that $A = X D X^{-1}$.

9. Let $A = \begin{pmatrix} 3 & 4 \\ -2 & -3 \end{pmatrix}$. Find e^A.