MAC 2313 (Multivariable Calculus)
Homework for chapter 11

Remember to do some of the following problems everyday. Be sure to have completed them before Test 1.

1. Describe the given surface; if it is a sphere, state its radius and center. If it is a point, state its coordinates. a) $x^2+y^2+z^2+6x-2y-6=0$. b) $x^2+y^2+z^2-2mx-6y-8z+50=0$, where m is a parameter. (discuss according to the values of m.)

2. a) Find an equation for the sphere passing through the origin and centered at the point $C(1,-2,5)$. b) Decide whether the points $A(2,3,1)$, $B(-1,1,-2)$ and $C(1,-1,1)$ are the vertices of an equilateral triangle.

3. Let $\overrightarrow{r}$=2$\overrightarrow{i}$-3$\overrightarrow{j}$+4$\overrightarrow{k}$, $\overrightarrow{z}$=3$\overrightarrow{j}$-5$\overrightarrow{k}$, and $\overrightarrow{v}$=-2$\overrightarrow{i}$+$\overrightarrow{j}$-4$\overrightarrow{k}$. a) Find the area of the parallelogram having $\overrightarrow{r}$ and $\overrightarrow{z}$ as adjacent sides. b) Find the volume of the parallelepiped having $\overrightarrow{r}$, $\overrightarrow{z}$ and $\overrightarrow{v}$ as adjacent edges. c) Find the acute angle θ between $\overrightarrow{v}$ and the plane containing the face determined by $\overrightarrow{r}$ and $\overrightarrow{z}$.

4. Consider the lines: $L_1 : x=4-2t, \quad y=2+3t, \quad z=1+t$ and $L_2 : x=2+4t, \quad y=3-6t, \quad z=-2t$. a) Show that L_1 and L_2 are parallel lines. b) Find the distance between L_1 and L_2.

5. a) Let $A(x_0,y_0,z_0)$ be a given point in 3-space. Let $\mathcal{P}$ be the plane with equation $ax+by+cz+d=0$. Write down the distance D between A and the plane $\mathcal{P}$. $D=$
b) Use a) to find the distance between the two skew lines: $L_1 : x=-2+t, \quad y=3+2t, \quad z=1+8t$ and $L_2 : x=1-2t, \quad y=-2+3t, \quad z=-1+5t$.

6. Let $\overrightarrow{w}$=$\overrightarrow{i}$-2$\overrightarrow{j}$+3$\overrightarrow{k}$ and $\overrightarrow{v}$=2$\overrightarrow{i}$-$\overrightarrow{j}$-5$\overrightarrow{k}$. a) Find the vector component of $\overrightarrow{v}$ that is parallel to $\overrightarrow{w}$ and the vector component of $\overrightarrow{v}$ that is orthogonal to $\overrightarrow{w}$. b) If θ denotes the angle between $\overrightarrow{v}$ and $\overrightarrow{w}$, find $\cos(\theta)$ and $\sin(\theta)$. Is θ acute or obtuse? c) Find the direction angles of $\overrightarrow{w}$.

7. a) Set $\overrightarrow{u}$=$\overrightarrow{i}$-3$\overrightarrow{k}$, $\overrightarrow{v}$=-$\overrightarrow{j}$+$\overrightarrow{k}$ and $\overrightarrow{w}$=2$\overrightarrow{i}$-$\overrightarrow{j}$. Let $\overrightarrow{z}$=$\overrightarrow{i}$-$\overrightarrow{j}$+2$\overrightarrow{k}$. Find scalars a, b, and c such that $\overrightarrow{z}=a\overrightarrow{u}+b\overrightarrow{v}+c\overrightarrow{w}$. b) If we now set: $\overrightarrow{u}$=$\overrightarrow{i}$+$\overrightarrow{j}$-2$\overrightarrow{k}$, $\overrightarrow{v}$=$\overrightarrow{i}$+$\overrightarrow{j}$+$\overrightarrow{k}$ and $\overrightarrow{w}$=$\overrightarrow{i}$-$\overrightarrow{j}$, find scalars α, β and γ such that $\overrightarrow{z}=\alpha\overrightarrow{u}+\beta\overrightarrow{v}+\gamma\overrightarrow{w}$.

8. a) Find parametric equations for the line through the points $A(-1,2,3)$ and $B(2,-3,4)$. b) Find the vector $\overrightarrow{w}$ of norm 4 that is oppositely directed to $\overrightarrow{z}$=2$\overrightarrow{i}$-$\overrightarrow{j}$+3$\overrightarrow{k}$. c) Find parametric equations for the line through the point $A(5,0,-2)$ that is parallel to the planes $x-4y+2z=2$ and $2x+3y-z+1=0$. d) Find an equation for the plane through the points $A(-2,1,4)$, $B(1,0,3)$ that is perpendicular to the plane $4x-y+3z=-1$. c) Let L be the line defined by the parametric equations $x=1-2t$, $y=2+3t$, $z=3+t$. Let $\mathcal{P}$ be the plane defined by $2x+y-z=4$. c1) Show that L and $\mathcal{P}$ are not perpendicular to each other. c2) Find an equation for the plane $\mathcal{Q}$ that both contains L and is perpendicular to $\mathcal{P}$.

9. a) Show that the two lines $L_1 : x=1-t, \quad y=2+t, \quad z=1+5t$, and $L_2 : x=2+t, \quad y=2+3t, \quad z=-1+7t$ intersect, and find their point of intersection A. b) Find the acute angle θ between L_1 and L_2 at A. c) Find an equation for the plane that contains both L_1 and L_2. d) Find an equation for the plane that contains both L_1 and the point $B(1,-2,-1)$.

10. a) Find an equation for the surface that results when the elliptic cone $4x^2+9y^2-25z^2=0$ is reflected about the plane: i) $x=0$, ii) $y=0$, iii) $z=0$, iv) $x=y$, v) $y=z$, vi) $z=x$. b) Find an equation for the surface that results when the hyperboloid of one sheet $x^2+4y^2-z^2=1$ is translated to the point $D(-1,2,-3)$.

11. Show that the two lines $L_1 : x=4-t, \quad y=6, \quad z=7+2t$, and $L_2 : x=1+7t, \quad y=3+t, \quad z=5-3$ are skew, and find the distance between them.

12. a) Find an equation for the plane $\mathcal{P}$ that contains the line $L : x=3t, \quad y=1+t, \quad z=2t$, and is parallel to the intersection of the planes $y+z=-1$ and $2x-y+z=6$. b) Show that the lines $L_1 : x=-2+t, \quad y=3+2t, \quad z=4-t$ and $L_2 : x=3-t, \quad y=4-2t, \quad z=t$ are parallel, and find an equation for the plane they determine. c) Find the distance between L_1 and L_2.

13. a) Convert from rectangular to cylindrical coordinates: i) $(4\sqrt{3},-4,-4)$, ii) $(-3,3,-1)$.
b) Convert from cylindrical to rectangular coordinates: i) $(4,\frac{\pi}{6},-2)$, ii) $(7,\frac{2\pi}{3},5)$.

c) Convert from rectangular to spherical coordinates: i) $(\sqrt{3}, 1, -2)$, ii) $(-1, 1, \sqrt{2})$.
d) Convert from spherical to rectangular coordinates: i) $(3, \frac{5\pi}{6}, \frac{2\pi}{3})$, ii) $(4, \frac{7\pi}{12}, \frac{\pi}{6})$
e) Convert from cylindrical to spherical coordinates: i) $(\sqrt{5}, \frac{3\pi}{4}, -3)$, ii) $(3, \frac{11\pi}{6}, -2\sqrt{3})$.
f) Convert from spherical to cylindrical coordinates: i) $(5, \frac{\pi}{4}, \frac{5\pi}{6})$, ii) $(4, \frac{\pi}{6}, \frac{\pi}{2})$.

14. Convert the given equation from a) cylindrical to rectangular coordinates: i) $r = 4\sin\theta$, ii) $r = z$, iii) $r^2\cos(2\theta) = z$
b) spherical to rectangular coordinates: i) $\theta = \frac{\pi}{3}$, ii) $\phi = \frac{\pi}{4}$, iii) $\rho = 2\sec\phi$, iv) $\rho\sin\phi = 2\cos\theta$, v) $\rho = 4\cos\phi$, vi) $\rho\sin\phi = \cot\phi$. c) Identify each surface.

15. a) Draw the vector $\overrightarrow{u}$ starting at the point $C(-1, 2, -3)$ and ending at the point $D(1, -3, 1)$. b) Find the vector $\overrightarrow{w}$ with norm 4 that is oppositely directed to $\overrightarrow{u}$. c) Find an equation for the sphere having the points $A(2, 3, 4)$ and $B(2, 1, -2)$ as endpoints of a diameter. d) Find two vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ such that $2\overrightarrow{u} - \overrightarrow{v} = \overrightarrow{i} - 3\overrightarrow{j} + \overrightarrow{k}$ and $3\overrightarrow{u} + 2\overrightarrow{v} = 5\overrightarrow{i} - \overrightarrow{j} + 6\overrightarrow{k}$ e) Describe the surface whose equation is given, according to the values of the parameter m: $x^2 + y^2 + z^2 - 4mx + 2y - 6z + 35 = 0$.

16. Where does the line $x = 1 + t$, $y = 3 - t$, $z = 2t$ intersect the cylinder $x^2 + y^2 = 16$.

17. a) Let A, B and C be three arbitrary points in the space. Show that: if $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel vectors, then $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel vectors as well. b) When do three points A, B, and C lie on the same line?

18. Decide whether the two planes are parallel, perpendicular or neither. a) $\mathcal{P}_1 : 3x - 2y + z = 4$ and $\mathcal{P}_2 : x + 2y + z = 7$, b) $\mathcal{Q}_1 : 2x + y + z = 5$ and $\mathcal{Q}_2 : 4x + 2y + 2z = 3$.

19. Let L be a line and $\mathcal{P}$ be a plane in 3-space. Let $\overrightarrow{u}$ be a vector parallel to L and $\overrightarrow{n}$ be a normal vector to $\mathcal{P}$. a) When are L and $\mathcal{P}$ parallel? b) When are L and $\mathcal{P}$ perpendicular?

20. a) Problems 52, 53 and 54, p. 784 in text. b) A force $\mathbf{F}$=4$\overrightarrow{i}$-6$\overrightarrow{j}+\overrightarrow{k}$ is applied to a point that moves 15 meters in the direction of the vector $\overrightarrow{i}+\overrightarrow{j}+\overrightarrow{k}$. How much work is done by $\mathbf{F}$ if the magnitude of $\mathbf{F}$ is in newtons?

21. If $\overrightarrow{u}$,$\overrightarrow{v}$ and $\overrightarrow{w}$ are three nonzero vectors with
a) $\overrightarrow{u} \cdot \overrightarrow{v} = \overrightarrow{u} \cdot \overrightarrow{w}$. Do we have $\overrightarrow{v}$=$\overrightarrow{w}$? Explain.
b) $\overrightarrow{u} \times \overrightarrow{v} = \overrightarrow{u} \times \overrightarrow{w}$. Do we have $\overrightarrow{v}$=$\overrightarrow{w}$? Explain.
c) $\overrightarrow{u} \cdot \overrightarrow{v} = ||\overrightarrow{u} \times \overrightarrow{v}||$, what can you say about the angle between $\overrightarrow{u}$ and $\overrightarrow{v}$?

22. a) Show that in 3-space, the distance d from a point P to the line L through the points A and B is given by

$$d = \frac{||\overrightarrow{AP} \times \overrightarrow{AB}||}{||\overrightarrow{AB}||}.$$

b) Use the result in a) to find the distance between the point $P(-1, 2, -3)$ and the line through the points $A(1, 2, 3)$ and $B(2, 3, 1)$.

23. Two bugs are walking along lines in 3-space. At time t bug 1 is at the point (x, y, z) on the line

$$x = 4 - t, \quad y = 1 + 2t, \quad z = 2 + t$$

and at the same time, bug 2 is at the point (x, y, z) on the line

$$x = t, \quad y = 1 + t, \quad z = 1 + 2t.$$

Assuming that the distance is in centimeters and time in minutes, how close do the bugs get,and at what time?