MAC 2312 (Calculus II) — Answers
QUIZ 10, Friday November 04, 2016

Name: **PID:**

Remember to show all your work; you won't get any credits if only your answers are shown without the steps leading to them.

1. [3] Find the Taylor series for the function f given by $f(x) = \frac{1}{x}$ about $x = -3$.

$f'(x) = -\frac{1}{x^2}$, $f''(x) = \frac{2}{x^3}$, $f^{(3)}(x) = -\frac{6}{x^4}$, so $f^{(k)}(x) = \frac{(-1)^k k!}{x^{k+1}}$

$f^{(k)}(-3) = \frac{(-1)^k k!}{(-3)^{k+1}}$

Taylor series: $\sum_{k=0}^{\infty} \frac{f^{(k)}(-3)}{k!}(x+3)^k = \sum_{k=0}^{\infty} \frac{(-1)^k k!(x+3)^k}{(-1)^{k+1}3^{k+1}k!} = -\sum_{k=0}^{\infty} \frac{(x+3)^k}{3^{k+1}}$

You may also use your knowledge of $\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$, and algebra:

$\frac{1}{x} = \frac{-1}{3-(x+3)} = \frac{-1/3}{1-\frac{(x+3)}{3}} = -\frac{1}{3}\sum_{k=0}^{\infty} \frac{(x+3)^k}{3^k} = -\sum_{k=0}^{\infty} \frac{(x+3)^k}{3^{k+1}}$

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2. [3] Find the MacLaurin series for the function g defined by $g(x) = e^{-2x}$.

$g'(x) = -2e^{-2x}$, $g''(x) = 4e^{-2x}$, $g^{(3)}(x) = -8e^{-2x}$, so $g^{(k)}(x) = (-1)^k 2^k e^{-2x}$

$g^{(k)}(0) = (-1)^k 2^k$

Maclaurin Series: $\sum_{k=0}^{\infty} \frac{g^{(k)}(0)}{k!} x^k = \sum_{k=0}^{\infty} \frac{(-1)^k 2^k x^k}{k!}$

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3. [4] Find the radius of convergence and the interval of convergence of the power series $\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{k+1}$.

$\rho = \lim_{k\to\infty} \frac{|(-1)^{k+1}x^{k+1}|}{k+2} \cdot \frac{k+1}{|(-1)^k x^k|}$

$= |x| \lim_{k\to\infty} \frac{k+1}{k+2} = |x| \lim_{k\to\infty} \frac{k(1+\frac{1}{k})}{k(1+\frac{2}{k})} = |x| < 1 \longrightarrow R = 1.$

Series converges absolutely on $(-1,1)$.

At $x = -1$: $\sum_{k=0}^{\infty} \frac{(-1)^k(-1)^k}{k+1} = \sum_{k=0}^{\infty} \frac{1}{k+1} = \sum_{k=1}^{\infty} \frac{1}{k}$, diverges; harmonic series

At $x = 1$: $\sum_{k=0}^{\infty} \frac{(-1)^k}{k+1}$, converges; A.S.T

Hence $I_C = (-1, 1]$