MAC 2313 (Multivariable Calculus) -- Answers
QUIZ 10, Friday November 04, 2016

Name: **PID:**

Remember that no documents or calculators, or any other electronic devices are allowed during the quiz. Also remember that you won't get any credit(s) if you do not show the steps to your answers.

1. [3] Evaluate the triple integral $\int_0^{\pi}\int_0^1\int_0^{x^3} xz\sin(y)\,dzdxdy$.

$$= \int_0^{\pi}\sin(y)dy \cdot \int_0^1 x\left[\frac{z^2}{2}\right]_0^{x^3}dx$$
$$= -\cos(y)\Big]_0^{\pi}\cdot\frac{1}{2}\int_0^1 x(x^6)dx$$
$$= (-\cos(\pi)+\cos(0))\,\frac{1}{2}\,\frac{x^8}{8}\Big]_0^1$$
$$= 2\left(\frac{1}{2}\right)\frac{1}{8}$$
$$= \frac{1}{8}$$

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2. [3] Find the volume of the solid bounded above by the paraboloid $z = 8 - x^2 - y^2$ and below by the paraboloid $z = x^2 + y^2$.

intersection of two surfaces: $8-x^2-y^2 = x^2+y^2 \longrightarrow 2(x^2+y^2) = 8 \longrightarrow x^2+y^2 = 4$

R = projection of solid on xy-plane $= \{(x,y);\ x^2+y^2 \le 4\}$

$$V = \iint_R \int_{x^2+y^2}^{8-x^2-y^2} dz\,dA = \int_0^{2\pi}\int_0^2\int_{r^2}^{8-r^2} r\,dz\,dr\,d\theta = 2\pi\int_0^2 r(8-2r^2)dr$$
$$= 4\pi\int_0^2 4r - r^3\,dr$$
$$= 4\pi\left[2r^2 - \frac{r^4}{4}\right]_0^2$$
$$= 4\pi(8-4)$$
$$= 16\pi$$

(↑ Cylindrical coordinates)

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3. [4] Convert the triple integral $\int_0^2\int_{-\sqrt{4-x^2}}^0\int_{\sqrt{x^2+y^2}}^{4-\sqrt{x^2+y^2}} \sin(x^2+y^2)\,dzdydx$ to spherical coordinates, including the appropriate limits of integration, but do not evaluate it.

$x = \rho\sin\phi\cos\theta$
$y = \rho\sin\phi\sin\theta$
$z = \rho\cos\phi$
$x^2+y^2 = \rho^2\sin^2\phi$
$\sqrt{x^2+y^2} = \rho\sin\phi$
$\sqrt{x^2+y^2} \le z \le 4-\sqrt{x^2+y^2}$
$\rho\sin\phi \le \rho\cos\phi \le 4-\rho\sin\phi$
$\tan\phi \le 1$ → $0 \le \phi \le \frac{\pi}{4}$
$\rho(\cos\phi+\sin\phi) \le 4$ → $\rho \le \frac{4}{\cos\phi+\sin\phi}$

If R = projection of solid involved on xy-plane,

$$\int_0^2\int_{-\sqrt{4-x^2}}^0\int_{\sqrt{x^2+y^2}}^{4-\sqrt{x^2+y^2}} \sin(x^2+y^2)\,dzdydx$$
$$= \int_{\frac{3\pi}{2}}^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\frac{4}{\cos\phi+\sin\phi}} \sin(\rho^2\sin^2\phi)\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$
$$= \int_{-\pi/2}^{0}\int_0^{\pi/4}\int_0^{\frac{4}{\cos\phi+\sin\phi}}$$