MAC 2312 (Calculus II) — Answers
QUIZ 11, Friday November 18, 2016

Name: **PID:**

Remember to show all your work where necessary; you won't get any credits if only your answers are shown without the steps leading to them.

1. [5] Write down the Maclaurin series, using the sigma notation, for

a) $\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$

b) $\frac{1}{1+x^2} = \sum_{k=0}^{\infty} (-1)^k x^{2k}$

c) $\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}$

d) $\cos(x) = \sum \frac{(-1)^k x^{2k}}{(2k)!}$

e) $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

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2. [5] a) Use the Maclaurin series: $\frac{1}{1+x} = \sum_{k=0}^{\infty}(-1)^k x^k$, valid for $-1 < x < 1$, to find the Maclaurin series for $f(x) = \frac{1}{1-4x}$. b) Find the derivative function f', and use the Maclaurin series in a) to find the Maclaurin series for the derivative $f'(x)$. c) Use a well-known Maclaurin series to evaluate the integral $\int_0^1 e^{x^3}\,dx$. Explain your answers in b) and c), and use the sigma notation.

a) $\frac{1}{1-4x} = \frac{1}{1+(-4x)} = \sum_{k=0}^{\infty} (-1)^k(-4x)^k = \sum_{k=0}^{\infty} (-1)^k(-1)^k 4^k x^k = \sum_{k=0}^{\infty} 4^k x^k$

b) $f'(x) = \frac{4}{(1-4x)^2} = \sum_{k=0}^{\infty} 4^k \frac{d}{dx}(x^k) = \sum_{k=1}^{\infty} k 4^k x^{k-1}$

Term by Term Differentiation Theorem

c) $e^{x^3} = \sum_{k=0}^{\infty} \frac{(x^3)^k}{k!}$; so $\int_0^1 e^{x^3}dx = \int_0^1 \sum_{k=0}^{\infty} \frac{x^{3k}}{k!}dx \overset{\text{Term by Term Integration Theorem}}{=} \sum_{k=0}^{\infty} \frac{1}{k!}\int_0^1 x^{3k}dx$

$= \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{x^{3k+1}}{3k+1}\right]_0^1$

$= \sum_{k=0}^{\infty} \frac{1}{k!(3k+1)}$.