MAP 2302 (Differential Equations) — Answers
QUIZ 1, Friday January 19, 2018

Name: **PID:**

1. [4] Find all numbers m such that the function $f(x) = e^{mx}$ is a solution of the differential equation: $y''' - 3y' + 2y = 0$.

$f'(x) = me^{mx}$, $f''(x) = m^2 e^{mx}$, $f'''(x) = m^3 e^{mx}$

$f'''(x) - 3f'(x) + 2f(x) = 0 \to (m^3 - 3m + 2)e^{mx} = 0$ for all x

So $m^3 - 3m + 2 = 0$, as $e^{mx} \neq 0$ for all m, x

Now $m^3 - 3m + 2 = (m-1)(m^2 + m - 2) = (m-1)(m-1)(m+2) = 0$

So $m = 1$ or $m = -2$.

Hence f is a solution of the D.E. for $m = 1$ or $m = -2$.

2. [6] Find constants c_1 and c_2 such that the function $g(x) = c_1 e^{2x} + c_2 e^{-3x}$ solves the initial-value problem: $\begin{cases} y'' + y' - 6y = 0 \\ y(0) = 2, \quad y'(0) = -3 \end{cases}$

$g'(x) = 2c_1 e^{2x} - 3c_2 e^{-3x}$, $g''(x) = 4c_1 e^{2x} + 9c_2 e^{-3x}$

$g''(x) + g'(x) - 6g(x) = 4c_1 e^{2x} + 9c_2 e^{-3x} + 2c_1 e^{2x} - 3c_2 e^{-3x} - 6c_1 e^{2x} - 6c_2 e^{-3x}$

$= (4c_1 + 2c_1 - 6c_1)e^{2x} + (9c_2 - 3c_2 - 6c_2)e^{-3x}$

$= 0$; so g solves the D.E

For g to solve IVP, we must also have $g(0) = 2$ and $g'(0) = -3$.

Therefore

$g(0) = c_1 + c_2 = 2 \quad$ (i)

$g'(0) = 2c_1 - 3c_2 = -3 \quad$ (ii)

$3 \cdot$ (i) + (ii) yields

$5c_1 = 6 - 3 = 3 \to c_1 = 3/5$

$-2 \cdot$ (i) + (ii) yields

$-2c_2 - 3c_2 = -7 \to -5c_2 = -7 \to c_2 = 7/5$.

Hence $g(x) = \frac{3}{5} e^{2x} + \frac{7}{5} e^{-3x}$ is the unique solution of the IVP.