MAC 2312 (Calculus II) — Answers
QUIZ 2, Friday September 2, 2016

Name: **PID:**

Show your work. Remember that you won't get any credit if you do not show the steps to your answers.

1. [2] Let $F(x) = \int_x^2 \frac{t^3}{t^2+t+4}\,dt$. Find $F(2)$ and $F'(2)$.

$F(2) = \int_2^2 \frac{t^3}{t^2+t+4}\,dt = 0$ 0.5 ; $F(x) = -\int_2^x \frac{t^3}{t^2+t+4}\,dt$ 0.5

$F'(x) = -\frac{x^3}{x^2+x+4}$ 0.5 , by FTC2; So $F'(2) = -\frac{8}{4+2+4} = -\frac{8}{10} = -\frac{4}{5}$ 0.5

2. [2] A particle moves along a straight line with velocity given in ft/s by $v(t) = \sin t$, for every $t \geq 0$. Find the distance traveled during the time interval $[\frac{\pi}{2}, \frac{3\pi}{2}]$.

$d = \int_{\pi/2}^{3\pi/2} |\sin t|\,dt$ 0.5 $= \int_{\pi/2}^{\pi} \sin t\,dt$ 0.25 $+ \int_{\pi}^{3\pi/2} -\sin t\,dt$ 0.25, as $\sin\pi = 0$, and $\sin t \geq 0$ on $[\frac{\pi}{2}, \pi]$, $\sin t \leq 0$ on $[\pi, \frac{3\pi}{2}]$

$= -\cos t\Big]_{\pi/2}^{\pi} + \cos t\Big]_{\pi}^{3\pi/2}$ 0.5

$= -(-1) + 0 + 0 - (-1)$ 0.25

$= 2$ 0.25

3. [2] If $g(x) = 3^x - \frac{4}{\cos^2 x}$, find the average value of g on $[0, \frac{\pi}{3}]$.

$g_{ave} = \frac{3}{\pi}\int_0^{\pi/3} (3^x - 4\sec^2 x)\,dx$ 0.5 $= \frac{3}{\pi}\left(\frac{3^x}{\ln 3} - 4\tan x\right)\Big]_0^{\pi/3}$ 1

$= \frac{3}{\pi}\left(\frac{3^{\frac{\pi}{3}}}{\ln 3} - 4\sqrt{3} - \left(\frac{1}{\ln 3} - 0\right)\right)$ 0.5

$= \frac{3}{\pi}\left(\frac{3^{\frac{\pi}{3}} - 1}{\ln 3} - 4\sqrt{3}\right)$

4. [4] Evaluate each integral:

a) $\int_1^2 \frac{2x^2+1}{x}\,dx = \int_1^2 \frac{2x^2}{x} + \frac{1}{x}\,dx$ 0.5

$= \int_1^2 2x + \frac{1}{x}\,dx$ 0.5

$= x^2 + \ln x\Big]_1^2$ 0.5

$= 4 + \ln 2 - (1 + \ln 1)$ 0.25

$= 3 + \ln 2$, as $\ln 1 = 0$ 0.25

b) $\int_0^1 \left(\frac{3}{1+t^2} - \frac{4}{\sqrt{1-t^2}}\right) dt =$

$= 3\arctan(t) - 4\arcsin(t)\Big]_0^1$ 1

$= 3\arctan(1) - 4\arcsin(1) - 0$ 0.5

$= 3\left(\frac{\pi}{4}\right) - 4\left(\frac{\pi}{2}\right)$ 0.5

$= \frac{3\pi}{4} - 2\pi = -\frac{5\pi}{4}$