MAC 2313 (Multivariable Calculus) —Answers
QUIZ 2, Friday September 2, 2016

Name: **PID:**

Remember that no documents or calculators, or any other electronic devices are allowed during the quiz. Also remember that you won't get any credit(s) if you do not show the steps to your answers.

1. [3] Find the volume V of the parallelepiped having the vectors $\vec{u}=\vec{i}+3\vec{j}+5\vec{k}$, $\vec{v}=2\vec{i}+\vec{j}+\vec{k}$ and $\vec{w}=\vec{i}-\vec{j}-\vec{k}$ as adjacent edges.

$V=|\vec{u}\cdot(\vec{v}\times\vec{w})|$. 0.5 Now $\vec{u}\cdot(\vec{v}\times\vec{w})=\begin{vmatrix}1 & 3 & 5\\ 2 & 1 & 1\\ 1 & -1 & -1\end{vmatrix}=1(-1+1)-3(-2-1)+5(-2-1)=9-15=-6$ 2

Hence
$V=|-6|=6$ 0.5

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2. [3] Let $\vec{s}=-2\vec{i}+3\vec{j}-4\vec{k}$, and $\vec{t}=\vec{i}-\vec{j}+2\vec{k}$. Find the vector component of $\vec{t}$ that is parallel to $\vec{s}$.

Component of $\vec{t}$ that is parallel to $\vec{s}$ = $\text{proj}_{\vec{s}}(\vec{t})=\dfrac{\vec{t}\cdot\vec{s}}{\|\vec{s}\|^2}\vec{s}$ 1

Now, $\vec{t}\cdot\vec{s}=-2(1)+3(-1)-4(2)=-13$ 0.5

$\|\vec{s}\|^2=4+9+16=29$ 0.5

Hence
$\text{Proj}_{\vec{s}}(\vec{t})=-\dfrac{13}{29}\vec{s}=\dfrac{26}{29}\vec{i}-\dfrac{39}{29}\vec{j}+\dfrac{52}{29}\vec{k}$ 0.5
0.5

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3. [4] Decide whether the statement is true or false. No explanation needed.

1 a) The area of the parallelogram having the vectors $\vec{u}$ and $\vec{v}$ as adjacent sides is $\|\vec{u}\times\vec{v}\|$. True, by Theorem 11.4.5

1 b) If $\vec{u}$ and $\vec{v}$ are any two orthogonal vectors, then $\|3\vec{u}-2\vec{v}\|^2=9\|\vec{u}\|^2+4\|\vec{v}\|^2$. True

1 c) If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are any three nonzero vectors, then the volume of the parallelepiped having $\vec{u}$, $\vec{v}$ and $\vec{w}$ as adjacent edges is given by $V=\vec{u}\cdot(\vec{v}\times\vec{w})$. False, $V=|\vec{u}\cdot(\vec{v}\times\vec{w})|$

1 d) If $\vec{u}$ and $\vec{v}$ are any two vectors, then $\vec{u}\times(\vec{v}-\vec{u})=\vec{u}\times\vec{v}$. True, as $\vec{u}\times(\vec{v}-\vec{u})=\vec{u}\times\vec{v}-\vec{u}\times\vec{u}$ and $\vec{u}\times\vec{u}=\vec{0}$

$$\|3\vec{u}-2\vec{v}\|^2=(3\vec{u}-2\vec{v})\cdot(3\vec{u}-2\vec{v})=3^2\|\vec{u}\|^2-6\,\vec{u}\cdot\vec{v}+2^2\|\vec{v}\|^2=9\|\vec{u}\|^2+4\|\vec{v}\|^2, \text{ as } \vec{u}\cdot\vec{v}=0$$