MAP 2302 (Differential Equations) — Answers
QUIZ 2, Friday January 26, 2018

Name: **PID:**

1. [4] Transform the differential equation $(x^2 + y^2)dx + x(x - y)dy = 0$ into a separable equation by setting $y = vx$. But, do not solve the separable equation obtained.

$dy = v\,dx + x\,dv$, D.E becomes

$(x^2 + x^2v^2)dx + (x^2 - x^2v)(v\,dx + x\,dv) = 0$

Grouping the terms involving dx:

$(x^2 + \cancel{x^2v^2} + x^2v - \cancel{x^2v^2})dx + x^3(1-v)dv = 0$

$x^2(1+v)dx + x^3(1-v)dv = 0$, which is a separable equation.

2. [6] Determine whether the differential equation $(e^x + y^2)dx + (2xy + e^{-y})dy = 0$ is exact. If it is exact, solve it. If not, explain why.

Set $M(x,y) = e^x + y^2$, $N(x,y) = 2xy + e^{-y}$

$\frac{\partial M}{\partial y}(x,y) = 2y = \frac{\partial N}{\partial x}(x,y)$; so D.E is exact.

Solution method 1: (Grouping)

$e^x dx + y^2dx + 2xy\,dy + e^{-y}dy = 0$

$d(e^x) + d(xy^2) + d(-e^{-y}) = 0$

$d(e^x + xy^2 - e^{-y}) = 0$; so general solution is

$e^x + xy^2 - e^{-y} = c$, c = constant

Solution method 2: (Standard). Since D.E is exact, there exists a function F such that

(i) $\frac{\partial F}{\partial x}(x,y) = e^x + y^2$

(ii) $\frac{\partial F}{\partial y}(x,y) = 2xy + e^{-y}$

Integrating (ii) w.r.t. y yields

(iii) $F(x,y) = \int (2xy + e^{-y})dy = xy^2 - e^{-y} + k(x)$

Differentiate (iii) w.r.t. x ~~to~~ get:

$\frac{\partial F}{\partial x}(x,y) = y^2 + k'(x)$
$= y^2 + e^x$, by (i)

Therefore $k'(x) = e^x$, so $k(x) = e^x + d$, d = constant

We may choose $d = 0$, so that $F(x,y) = xy^2 - e^{-y} + e^x$

General ~~sol~~ solution of D.E: $xy^2 - e^{-y} + e^x = c$, c = constant